Yellow = Vocab

Red = Key Concepts

Aug. 18. 2023

1.1 Functions

Warm-Up

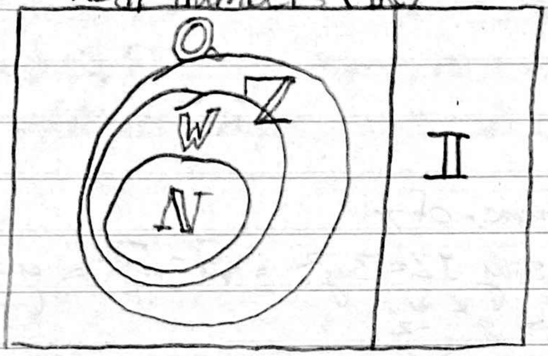
2÷ 4	4- 1	5	6× 2	8+ 3
2	8+ 4	1	3	5
15× 5	3	2÷ 2	1	20× 4
3	2÷ 2	4	5	1
4- 1	5	3 3	2- 4	2

"Ken-ken" ✓

Real Numbers

Letter	Set	Examples
$\mathbb{Q}$	rationals	$0.125, -\frac{7}{8}, 2/3 = 0.\overline{66}$
$\mathbb{I}$	irrationals	$\sqrt{3} = 1.73\ldots, \pi, e$
$\mathbb{Z}$	integers	$-5, 17, -23, 8$
$\mathbb{W}$	wholes	$0, 1, 2, 3\ldots$
$\mathbb{N}$	naturals	$1, 2, 3, 4\ldots$

Real Numbers ($\mathbb{R}$)



Set-Builder Notation Expression that describes a set of numbers

↗Element

ex: $\{x \mid x \geq 2,\ x \in \mathbb{N}\} = \{2, 3, 4, 5, \ldots\}$

"The set" "Such that" "Natrual numbers"

Interval Notation Another way to describe a set of numbers

[ ]: ≤, ≥, •

( ): <, >, ○ ±∞ only $\mathbb{R}$

Union When you have an "or" inequality - ∪

ex: $[-2, 5] \cup (12, \infty)$

Function Every x has only one y

⓪, 7, 14, 21, 28, 35, 42 ...

Example 1. A: $\{x \mid 2 \leq x \leq 7, x \in \mathbb{N}\}$

B: $\{x \mid x \geq -17, x \in \mathbb{R}\}$

C: $\{x \mid x = 7n, n \in \mathbb{I}\}$

Example 2: A: $[-2, \quad 12]$

B: $(-4, \infty)$

C: $(-\infty, 3) \cup [54, \infty]$

Example 3: D: $x = 3y^2 = y$ as a func. of x

not a function   say $12 = 3y^2 = \sqrt{4} = \sqrt{y^2} = y = \pm 2$

2 values → 2, -2

HW: pg. 19 #'s 1-45 EOO, 55, 61, 69
(omit # 5, 21, 29)

1.2 Notes Analyzing Graphs of Functions + Relations

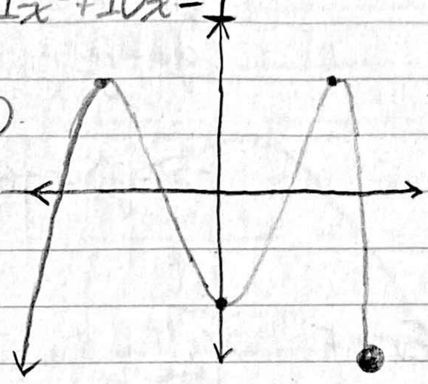
Aug. 21. 2023

Ex 1: A) 1,050,000 $-5(3)^2+50(3)=105$

B) $f(x)=-5x^2+50x \to 0=-5x^2+50x-5 = -1x^2+10x=1$

$125=-5x^2+50x \to 0=-5x^2+50x-125$

$-5(x^2+50x-125)$

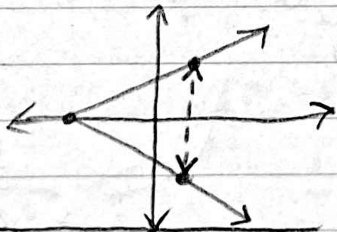
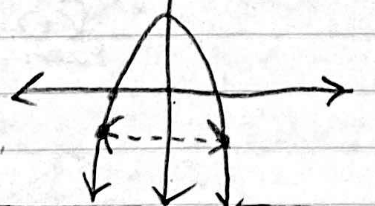
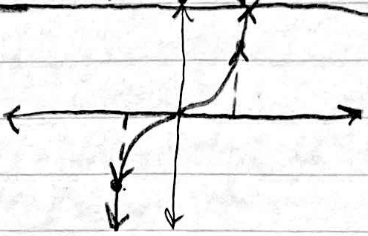


Ex 2: A) $\{x|x\le 3, x\in\mathbb{R}\}$ $\{y|y\le 2, y\in\mathbb{R}\}$

Ex 3: A) $f(x)=x^2-4x\boxed{+4}$ est: (0,4) alg: $0^2-4(0)+4=(0,4)$

Ex 4: A) est: $x=-1,0,1$ alg: $0=x^3-x \to x(x^2-1) \to x(x+1)(x-1)$ $x=0,\pm1$

Tests for Symm

Graphical Test		
x-axis		replace y w/ -y
y-axis		replace x w/ -x
origin		↑ both

Ex 5: A) $xy=-6$

x-axis

$y=-y$

~~$x(-y)=-6$~~

~~$\frac{-xy}{-1}=\frac{-6}{-1}$~~

~~$xy=6$~~

y-axis

~~$x=-x$~~

~~$\frac{-xy}{-1}=\frac{-6}{-1}$~~

~~$xy=6$~~

origin

$x=-x$

$y=-y$

$-x(-y)=-6$

Even Function: Functions are symmetric with respect to the y-axis

alg. test: $f(-x)=f(x)$

Odd Function: Functions that are symmetric with respect to the origin

alg. test: $f(-x)=-f(x)$

Ex6: C) $f(x)=x^3-3x^2-x+3$

$f(-x)$

$\swarrow \quad \searrow$

$\underset{\text{even}}{f(x)} \quad \underset{\text{odd}}{-f(x)}$

1-3

Aug 28. '23

Continuous Function:

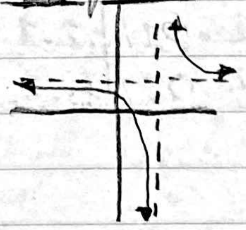
A function that has no breaks, holes, gaps.

Discontinuous Function:

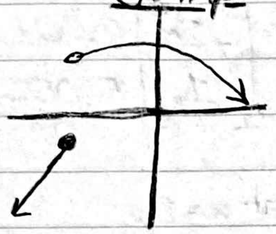
Function with holes, gaps, breaks

Types of discontinuity:

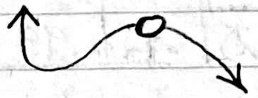
Infinite



Jump

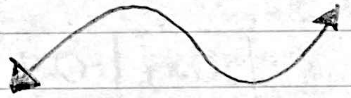


Removable



Limit:

The concept of approaching a value without ever reaching it



$\lim_{x \to \infty} f(x) = \underline{\infty}$

End behavior:

$x \to \infty$ | $x \to -\infty$

$f(x) \to \infty$ | $f(x) \to -\infty$

$\lim_{x \to -\infty} f(x) = \underline{-\infty}$

Same

Continuity Test:

1. Does $f(c)$ exist?
2. Does $\lim_{x \to c} f(x)$ exist?
3. Does $\lim_{x \to c} f(x) = f(c)$ exist?

c = specific point

← make table

step #1 = step #2?

Ex 1: A: $f(x) = 2|x| + 3$

c

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	6.8	6.98	6.998	7	7.002	7.02	7.2

Pick values extremely close to "c"!

And put in numerical order

① does $f(2)$ exist?

$2|2| + 3$

$4 + 3 = 7$ Yes

② Does $\lim_{x \to 2} f(x)$ exist?

Yes. $f(x) \to 7$.

③ Does $\lim_{x \to 2} f(x) = f(2)$ exist?

$7 = 7$, yes Continuous at $x = 2$

B : ① $f(x)=\frac{2x}{x^2-1}$ at $x=1$. $\frac{2(1)}{1^2-1} = \frac{2}{0}$ = undefined!

②

x	.9	.99	.999	1^c	1.001	1.01	1.1
$f(x)$	-9.47	-99.5	-999.5	und.	1000.5	100.5	10.47
				$-\infty$ ∞			

Round to largest needed value

③ Infinite discontinuity at $x=1$.

C : ① $f(2)=1$, $2-1=1$ ¦ defined

②

x	1.9	1.99	1.999	2^c	2.001	2.01	2.1
$f(x)$	.9	.99	.999	1	5.002	5.02	5.2
				1 5			

③ Jump discontinuity at $x=2$.

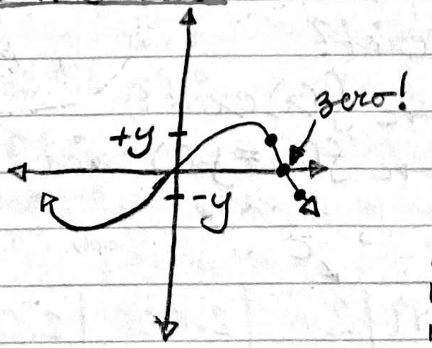
D : ① $f(-1)$ = und. ¦ $f(x)=\frac{x+1}{x^2+3x+2}$

②

x	-1.1	-1.01	-1.001	-1^c	-0.999	-0.99	-0.9
$f(x)$	1.111	1.01	1.001	und.	0.999	0.99	0.9
				1 1			

③ Removable discontinuity at $x=-1$.

<u>Intermediate value theorem</u> "IVT":



Ex 3: $f(x)=x^2-x-\frac{3}{4}$

$-1<x<0$ } zero #1
$(-1,0)$ }

$1<x<2$ } zero #2
$(1,2)$ }

★ Ex4: Use limit notation!

p40 # 5, 7, 9, 15-30(x5), 39-48(x3)
p772 # 1-4

1.4 Extrema and Avg. Rates of Change

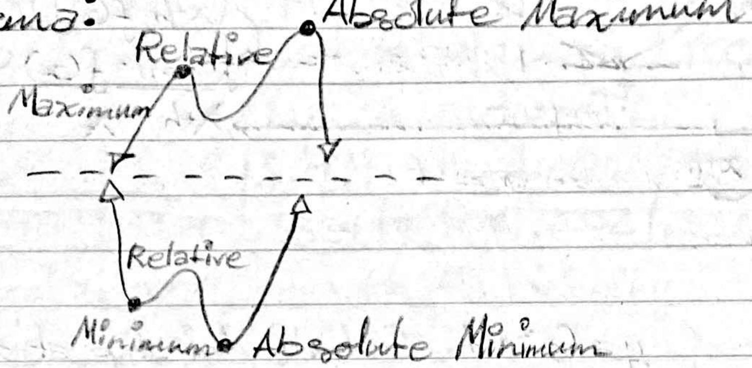
Aug. 30. '23

Ex 1: Dec: $(-\infty, 0)$ Inc: $(0, \infty)$

Critical points:

- Extrema (max & mins)
- Points of inflection

★ Extrema:



Ex 2:

Relative minimum of -7.931 at $x \approx -1.232$.
Relative maximum of 14.57 at $x \approx 2.151$.

complete homework with both x and y values
Round/truncate to 3rd decimal place

Average Rate of Change:

Slope between two points on curve

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Secant line:

Line through 2 points on a curve

Ex 5:

A: $y = -2x^2 + 4x + 6$; $[\overset{x \text{ values}}{-3, -1}]$

$$\begin{matrix}(-3,-24)\\(-1,0)\end{matrix} \xrightarrow[\text{form}]{\text{slope}} \frac{0+24}{-1+3} \quad \frac{24}{2} = \boxed{\frac{12}{1}} = 12$$

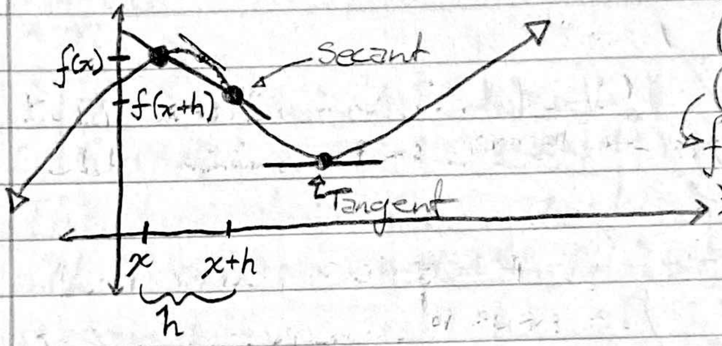
slope

12-4
ex 1

Derivative:

Using a limit to determine the slope of a line tangent to a graph of a function at any point

↳ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$



$(x, f(x))$

$(x+h, f(x+h))$

$\to \frac{f(x+h)-f(x)}{x+h-x} \to \overset{\text{approach 0 (where secant points meet)}}{\lim_{h\to 0}} \frac{f(x+h-f(x))}{h}$

Ex 1:

Derivative of $y=6x^2+7$. Evaluate derivative at $x=2,5$

$\lim_{h\to 0}\frac{6(x+h)^2+7-(6x^2+7)}{h}$

$\lim_{h\to 0}\frac{\cancel{6x^2}+12xh+6h^2+\cancel{7}-\cancel{6x^2}-\cancel{7}}{h}$

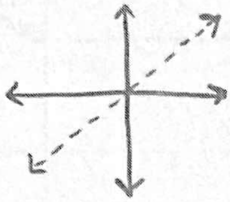
$\lim_{h\to 0}\frac{\cancel{h}(12x+6h)}{\cancel{h}}$ $h=0$ $12x+6\overset{(h=0)}{h}$ ← Evaluate

$12x+0=f'(x)$

1-5 Parent Functions and Transformations

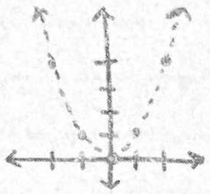
Sep. 8. 2023

Identity



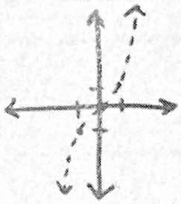
$f(x)=x$

Quadratic



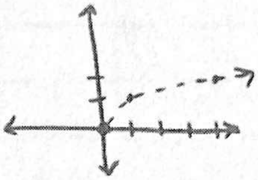
$f(x)=x^2$

Cubic



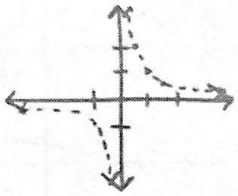
$f(x)=x^3$

Square Root



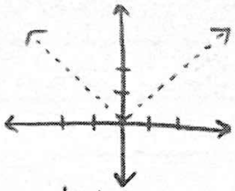
$f(x)=\sqrt{x}$

Reciprocal



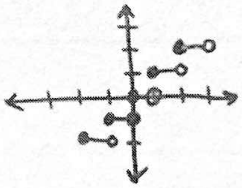
$f(x)=\frac{1}{x}$

Absolute Value



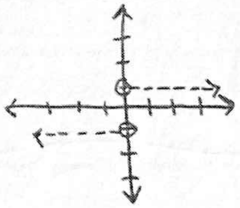
$f(x)=|x|$

Greatest Integer



$f(x)=[\![x]\!]$

$\frac{x}{|x|}=\frac{|x|}{x}$



Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(-\infty,0)\cup(0,\infty)$
~~In~~tercepts: none
Symm.: Origin $\longrightarrow$ Test: $x=-x$, $y=-y$ $\quad -y=\frac{1}{-x}$ $\quad \frac{-y}{-1}=\frac{-\frac{1}{x}}{-1}$ $\rightarrow y=\frac{1}{x}$ ✓
Continuity: $(-\infty,0)\cup(0,\infty)$
End behavior: $\lim_{x\to\infty} f(x)=0$ $\quad \lim_{x\to-\infty} f(x)=0$

Intervals (In/De):
Decreasing: $(-\infty,0)\cup(0,\infty)$

} Reciprocal $f(x)=\frac{1}{x}$

Transformation:
Affects appearance of the parent graph

Translation:
A vertical or horizontal shift

$f(x)+k$	$f(x-h)$
ex: $\sqrt{x}+2$	ex: $\sqrt{x+3}$
$k=2$	$h=-3$

} h always opposite

Reflection:

Mirror image in respect to an axis

x-axis y-axis

$-f(x$ $f(-x)$

ex: $-\sqrt{x}$ ex: $\sqrt{-x}$

Vertical Dilation	**Horizontal Dilation**
$a \cdot f(x)$	$f(cx)$
$\lvert a\rvert > 1$: expand (skinny)	$\lvert c\rvert > 1$: compress (~~skinny~~) fat
$0 < \lvert a\rvert < 1$: compress (fat)	$0 < \lvert c\rvert < 1$: expand (~~fat~~) Skinny

1.) Create a parent function table
2.) Factor out a 'c' if needed $\longrightarrow \sqrt{2x-6} \rightarrow \sqrt{2(x-3)}$
3.) Multiply by any dilations/reflections
   * Be careful to use the reciprocal for your 'c' value
     ↳ a: multiply to y-val. only
     c: $\frac{1}{c}$ mult. to x-val. only
4.) Add any translations
   ↳ h: add to x-values
   k: add to y-values

} Tables & Transformations

} reflects to opposite → Ex: "a" reflects x-axis

Ex4: $g(x) = -3|2x-8|+1$, find parent function $\overset{f(x)}{}$, compare

①

x	f(x)
$(\frac{1}{2})-2$	$2(-3)$
$(\frac{1}{2})-1$	$1(-3)$
$(\frac{1}{2})0$	$0(-3)$
$(\frac{1}{2})1$	$1(-3)$
$(\frac{1}{2})2$	$2(-3)$

② $g(x) = -3|2(x-4)|+1$

③ $a = -3$

$c = 2 \rightarrow (\frac{1}{2})$

④ $h = 4$

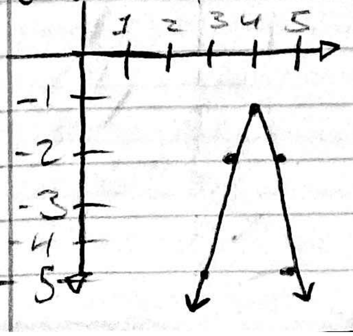
$k = 1$

→

x	f(x)
-1^{+4}	-6^{+1}
$-\frac{1}{2}^{+4}$	-3^{+1}
0^{+4}	0^{+1}
$\frac{1}{2}^{+4}$	-3^{+1}
1^{+4}	-6^{+1}

→

x	f(x)
3	-5
$3\frac{1}{2}$	-2
4	1
$4\frac{1}{2}$	-2
5	-5



$f(x)$ of $g(x) = \frac{1}{2}\sqrt{\frac{1}{3}x} - 1$

①

x	$f(x)$
0	0
1	1
4	2
9	3
16	4

② $\frac{1}{2}\sqrt{x \div 3} - 1$

③ $a: \frac{1}{2}$

$c: \frac{1}{3} \rightarrow \frac{3}{1} \rightarrow 3$

④ $h = 0$

$k = -1$

x	$f(x)$
0	0 -1
3	½ -1
12	1 -1
27	1½ -1
48	2 -1

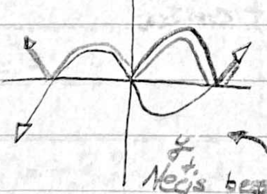
→

x	$f(x)$
0	-1
3	-½
12	0
27	½
48	1

Ex 5: $f(x) \begin{cases} |x+2| \text{ if } x < 0 \\ |x| - 2 \text{ if } 0 \leq x \leq 2 \\ \sqrt{x-2} \text{ if } x > 2 \end{cases}$

Transformations with Absolute Value:

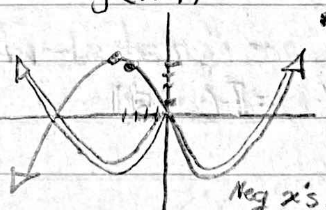
$|f(x)|$



y

Neg's become positive

$f(|x|)$



Neg x's disappear

reflect from positive x

x	$f(x)$
1	1
2	2
3	3
4	4
5	5

x	$f(x)$
-4	4
-5	5

1.6 Func. Operations + Composition

Sep. 11. 2023

Operations:

Sum	$(f+g)(x)$
Diff	$(f-g)(x)$
Product	$(f\cdot g)(x)$
Quotient	$\left(\frac{f}{g}\right)(x)$

Ex 1:

$f(x)=x^2-2x$ $g(x)=3x-4$

$h(x)=-2x^2+1$

A: $(f+g)(x)$

$x^2-2x+3x-4$

$=x^2+1x-4$

Dom: $(-\infty,\infty)$

B: $(f-h)(x)$

$x^2-2x+2x^2-1$

$=3x^2-2x-1$

Dom $(-\infty,\infty)$

C: $(f\cdot g)(x)$

$(x^2-2x)(3x-4)$

$3x^3-6x^2-4x^2+8x$

$=3x^3-10x^2+8x$

Dom $(-\infty,\infty)$

D: $\left(\frac{h}{f}\right)(x)$

$\frac{-2x^2+1}{x^2-2x} \rightarrow \frac{x^2-2x}{x(x-2)} \nearrow$ Dom $\mathbb{R}, x=0,2$

$\{x \mid x\neq 0,2, x\in\mathbb{R}\}$

Comp: ↙ As in questions

$[f\circ g](x)=f[g(x)]$ ⤵ Same!

$(f\circ g)(x)=f(g(x))$

$(x+3)(x+3)$ $x^2+\overbrace{3x+3x}^{6x}+9$

Ex 2:

$f(x)=2x^2-1$ $g(x)=x+3$

A: $[f\circ g](x)=f(g(x))$

$2(x+3)^2-1$

$2x^2+12x+18-1$

$\boxed{2x^2+12x+17}$

B: $[g\circ f](x)$

$(2x^2-1)+3$

$\boxed{2x^2+2}$

C: $[f\circ g](2)$

$f(g(5))$

$\boxed{49}$

Comp. w/ restricted domains

1. Analyze f(x) dom.
2. Analyze g(x) dom.
3. f∘g, analyze dom.
4. Pick most restricted dom.!

① $x-1\geq 0,$

$x\geq 1$ ④

$\{x \mid x\geq 2, x\in\mathbb{R}\}$

② $\mathbb{R}$

Ex 3:

A: f∘g: $f(x)=\sqrt{x-1}$, $g(x)=(x-1)^2$

$x^2-x-x+1$

x^2-2x+1

③ $\sqrt{(x-1)^2-1}$

$\sqrt{x^2-2x}$ = f∘g

$x^2-2x\geq 0$ $x\geq 2$

$x(x-2)\geq 0$ $x\leq 0$

$(f \circ g)(x)$

$=$

Ex 4:

A: $h(x) = \frac{1}{(x+2)^2}$

$f(x)$	$g(x)$
$\frac{1}{x}$	$(x+2)^2$
$\frac{1}{x^2}$	$x+2$

Sep. 13
2023

1.7 Inverse Relations and Functions

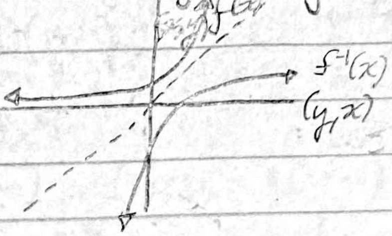
Inverse Relation:

One relation contains (x, y), the other (y, x)

Inverse Function:

Inverse relation, but also is a function. Use f and f^{-1}.

Reflects across $y=x$ (identity line)



One-to-one

Each x-value has 1 y-value

And each y-value has 1 x-value

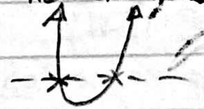
Horizontal Line Test:

Tells whether a function has an inverse WHICH is also a function

Ex: $|x+5|$ x^2+5

NOT one-to-one

Not one-to-one



Find Inverse Functions:

1 Horizontal line test
2 Switch x and y
3 Solve for y
4 Switch domain & range

Ex 2: A:

$f(x)=\frac{4-x}{x}, x\neq 0, y\neq -1$

① ✓ ② $x=\frac{4-y}{y}$

③ $xy=4-y$

$xy+y=4$

$y(x+1)=4$
GCF*

$y=\frac{4}{x+1}$

$f^{-1}(x)=\frac{4}{x+1}$

④ D: $x\neq -1$

R: $y\neq 0$

Composition of Inverse Functions:

Only inverses IF: $f\circ g=x$ AND $g\circ f=x$

Ex 3: $f(x)=\frac{2}{3}x+2$ and $g(x)=\frac{3}{2}(x-2)$, show they're inverse

$\overset{1}{f \circ g}$

$\frac{2}{3}\left(\frac{3}{2}(x-2)\right)+2$

$x-\cancel{2}+\cancel{2}$

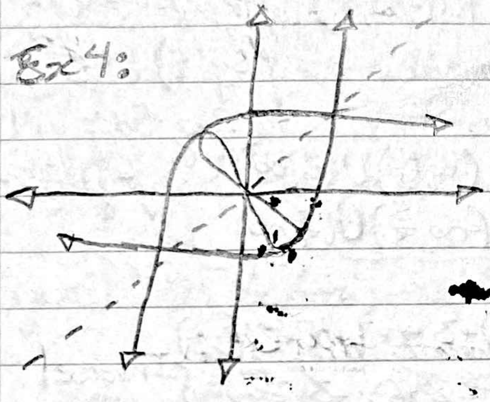
x ✓ ← Simplify to identity = inverse,

$g \circ f$

$\frac{3}{2}\left(\left(\frac{2}{3}x+\cancel{2}\right)-\cancel{2}\right)$

$\rightarrow \cancel{\frac{3}{2}}\left(\cancel{\frac{2}{3}}x\right)=x$ ✓

Ex 4:



Ex 5: Ⓐ $96{,}000+80x=f(x)$

$x=\frac{f(x)-96{,}000}{80}$

Ⓑ $x=100{,}000$,

$96{,}000+80(100{,}000)$

HW due Monday

Sep. 18
2023

2.1 Power & Radical Functions

Power Function

$f(x)=ax^n$, where "a" and "n" are non-zero, real numbers

Monomial Function

$f(x)=ax^n$, $n=$ positive integer

Ex 2: ⓐ $f(x)=2x^{-4}$ ← Analyze $2\boxed{x^{-4}}=\frac{2}{x^4}$

D: $(-\infty,0)\cup(0,\infty)$ R: $(0,\infty)$

x-int: none y-int: none

end behavior: $\lim_{x\to\infty} f(x)=0$ $\lim_{x\to-\infty} f(x)=0$

continuity: discontinuous at $x=0$

increasing: $(-\infty,0)$

decreasing: $(0,\infty)$

Ex 3: ⓑ $f(x)=4x^{-\frac{2}{5}}$ ex: → $\sqrt[2]{x^1}=x^{1/2}$ → $f(x)=\sqrt[5]{4x^{-2}}$

R: $\{y \mid y\neq 0, y\in\mathbb{R}\}$ D: $\{x \mid x\neq 0, x\in\mathbb{R}\}$

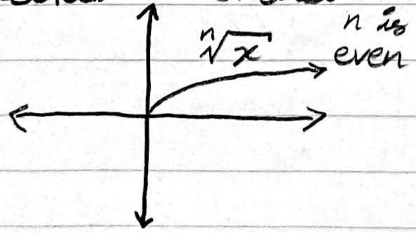
Int: none

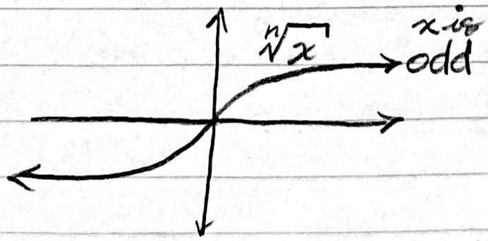
End behavior: $\lim_{x\to\infty} f(x)=0$ $\lim_{x\to-\infty} f(x)=0$

Cont.: $x\neq 0$ → $(-\infty,0)\cup(0,\infty)$

In.: $(-\infty,0)$ De.: $(0,\infty)$

Radical Functions:





extraneous solutions:

Solutions that do not satisfy original equation

Ex 6: ⓐ $2x=6\sqrt{20x+36}$

FOIL → $(2x-6)^2=\sqrt{20x+36}^{\,2}$

$4x^2-12x-12x\cancel{+36}=20x\cancel{+36}$

$4x^2-44x=0$

$4x(x-11)=0$ $\qquad \cancel{x=\cancel{0},11}$

check

$0: 2(0)=6+\sqrt{20(0)+36}$

$\cancel{0=12}$

$\boxed{x=11}$

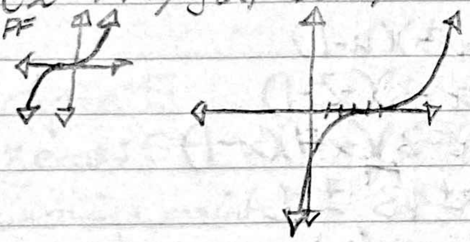
2023
Sep. 25

2.2 Polynomial Functions

Polynomial Function: $f(x)=\overset{\text{Leading coefficient}}{\overset{\downarrow}{a}}x^{n}+bx^{n-1}\ldots+cx^{2}+dx+\underbrace{ex^{0} \text{ or } e}$

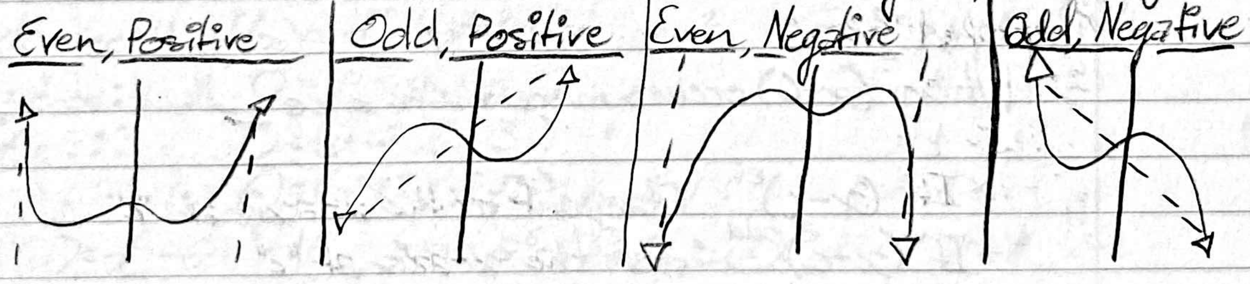
- continuous for all real numbers, smooth + rounded curves
- degree decreases numerically, regularly
- Whether your leading coefficient is + or − determines end behavior
  - Number in front of largest degreed monomial

Ex 1: A) $f(x)=(x-5)^{5}$



Leading term detemines:
End Behavior

ex: $f(x)=\boxed{-3}x^{2}+x+2$ ← Leading coefficient
- Neg coe, even degree



Odd/Even degree, Pos./Neg. Coefficient

Turning point:
When graph goes from increasing to de. + vice versa
Ex: Extrema

Zeros + Turning Points:
n = largest polynomial degree, n = zero count, $n-1$ = possible turning points

Ex: 3: A) $x^{3}+5x^{2}+4x$
Possible zeros: 3
T.P.: 2 (possible)

$0=x^{3}+5x^{2}+4x$
$0=x(x^{2}+5x+4)$
$0=x((x+1)(x+4))$
$x=0,-4,-1$

x^{10} x^5

x^8 x^4

x^4 x^2

Quadratic Form: au^2+bu+c

↳ second is $\frac{1}{2}$ of the first

Ex: $a(u)^2+b(u)+c$, $x^5=u$

Ex 4: A) $h(x)=x^4-4x^2+3$

degree: 4

zeros: 4

T.P.: 3

$0=x^4-4x^2+3$

$u=x^2$

$0=1(x^2)^2-4(x^2)+3$

u^2-4u+3

$0=(u-3)(u-1)$

$0=(x^2-3)(x^2-1)$

$0=(x^2-3)(x+1)(x-1)$

$x=\pm\sqrt{3}, \pm 1$

Repeated Zero:

When $(x-c)$ occurs more than once

*If $(x-c)^{\text{even}}$: tangent to the x-axis at "c"

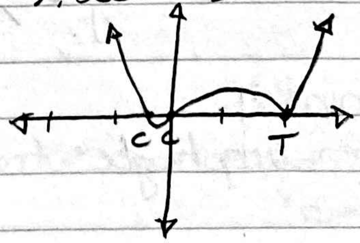
*If $(x-c)^{\text{odd}}$: crosses the x-axis at "c"

Ex 6: A)

$f(x)=x(3x+1)\boxed{(x-2)^2} \rightarrow (x-c)^2$, occurs 2 times

degree $= x^1\cdot(3x)^1\cdot(x)^2 = 3x^4$

zeros $= \underset{C}{\underset{\cdot 1}{0}}, \underset{C}{\underset{\cdot 1}{-\frac{1}{3}}}, \underset{T}{\underset{\cdot 2}{2}}$



---

*r
correlation coefficient
[-1,1]

*r^2
[0,1]

STAT 1: EDIT L_1/L_2

SCATPLOT

y= ON ⋰ L_1/L_2 ZOOM 9:ZOOM Stat

STAT → CALC → Reg Store in y_1

Sep 27
2023

2.3 Remainder + Factor Theorems

Ex 1: A) $6x^3+17x^2-104x+60$ → $(2x-5)$ is a factor

$$2x-5\overline{)6x^3+17x^2-104x+60}$$
quotient: $3x^2+16x-12$

$-6x^3+15x^2$ ↓

$32x^2-104x$

$-32x^2+80x$

$-24x+60$

$-(-24x+60)$

0 ✓

→ $(3x^2+16x-12)(2x-5)$

$x^2+16x-36$

$(x+18)(x-2)$

$(x+6)(3x-2)(2x-5)$

Ex 2: A)

$$2x-1\overline{)6x^3-5x^2+9x+6}$$
quotient: $3x^2-x+4+\left(\frac{10}{2x-1}\right)$

$-6x^3+3x^2$ ↓

$-2x^2+9x$

$-(+2x^2+1x)$

$8x\ 6$

$-(8x+4)$

10

Syth. Division:

Shortcut for polynomial div. by linear factor in $x-c$:

- * (x^1-c) → factor's var. degree is 1!
  - → LC must be 1
  - → insert "0" for missing terms
  - → _| ← zero

3	$2x^5$	$-4x^4$	$-3x^3$	$-6x^2$	$-5x$	-8
	↓	6	6	9	9	12
	2	2	3	3	4	R 4
	x^4	x^3	x^2	x^1	c	

→ $2x^4+2x^3+3x^2+3x+4+\frac{4}{x-3}$

$-\frac{1}{4}$	$\frac{8}{4}$	$\frac{38}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
	↓	$-\frac{2}{4}$	$-\frac{9}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$
	2	9	-1	1	R $\frac{2}{4}$
	x^3	x^2	x	c	

→ $\frac{\frac{2}{4}\cdot 4}{x+\frac{1}{4}\cdot 4}$

$2x^3+9x^2-x+1+\frac{2}{4x+1}$ or $\frac{\frac{2}{4}}{x+\frac{1}{4}}$

Remainder Theorem: If a polynomial $f(x)$ is divided by $x-c$, the the remainder is $f(c)$.

$(x-2) \rightarrow f(2)=\text{remainder}=f(c)$

Ex 5: $800+15x$ $600-10x$ $\leftarrow x=5$

$R(x)=(800+15x)(600-10x)$

$480000-8000x+9000x-150x^2$

$-150x^2+1000x+480000$

$$\begin{array}{r|rrr}
5 & -150 & +1000 & +480000 \\
 & \downarrow & -750 & 1250 \\
\hline
 & -150 & 250 & \boxed{R\ \underline{481,250}}
\end{array} \rightarrow R(5)$$

Factor Theorem: Polynomial $f(x)$ has a factor $(x-c)$ ONLY IF $f(c)=0$.

Ex 6:

$$\begin{array}{r|rrrr}
5 & 1 & -18 & 60 & 25 \\
 & \downarrow & 5 & -65 & -25 \\
\hline
 & 1 & -13 & -5 & R\ 0 \checkmark
\end{array}
\qquad
\begin{array}{r|rrrr}
-5 & 1 & -18 & 60 & 25 \\
 & \downarrow & -5 & 115 & -\ldots \\
\hline
 & 1 & -23 & 175 & R\ \times
\end{array} \text{ no factor!}$$

$\downarrow$

$(x-5)(x^2-13x-5)$

2.4 $\overset{\text{part a}}{}$ Zeros + Polynomial Functions

2.Oct.2023

Rational zero theorem: $\downarrow$ Non-calc on test

If polynomial $f(x)=qx^n+x^{n-1}\ldots x+p$ then all possible rational zeros are: $\pm\frac{\text{factors } p}{\text{factors } q}$

$q=$ L.T. coefficient $p=$ constant

Ex 1: $f(x)=x^3-3x^2-2x+4$

a) List p/q: $\pm\frac{1,2,4}{1}=\pm1,\pm2,\pm4$

$\underline{1}|$ $1\ \ -3\ \ -2\ \ 4$ $\rightarrow x^2-2x-4$

$\downarrow\ \ \ 1\ \ -2\ \ -4$ ✓ $(x-\ \ )(x-\ \ )$

$1\ \ -2\ \ -4\ |\ ^{R}0$ ✓ $\underline{x=1 \leftarrow \text{only zero}}$

Ex 3:

$f(x)=x^3+4x^2-2x+7$ $\qquad \pm\frac{1,3}{1}\rightarrow\pm1,\pm3$

$10=x^3+4x^2-2x+7$

$0=x^3+4x^2-2x-3$ $\nearrow$

Use calc, find zero, test with synth. div.

Ex 6: $x=-1,\ 2,\ 2-i,\ 2+i$ $\qquad \rightarrow i^2=-1$

$(x+1)(x-1)[x-(2-i)][x-(2+i)]$

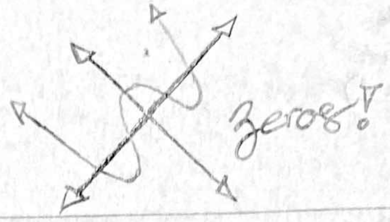
$(x-2+i)(x-2-i)$

Fundamental Theorem of Algebra: Polynomial func. of degree "n" has at least one zero (real/imaginary) in the complex number system

Linear Factorization Theorem: If $f(x)$ is a polynomial function of degree "n" >0, then f has exactly "n" linear factors including real, complex, and repeated zeros

Conjugate Root Theorem: If a polynomials root is "a+bi", then it's a complex conjugate, and "a−bi" is also a root.

Oct.4.2023



2.4 part b

☆ Linear Factorization Theorem:
If poly. func. is degree "$n > 0$", func. has "n" linear factors with real, complex, and repeated zeros
↳ Must only contain factors of degree 1

☆ Conjugate Root Theorem: If a root is in the form "a+bi", then it's a complex conjugate, and "a-bi" is also a factor.
↳ Think quadratic formula

$x = -1, 2, 2-i, 2+i$

$(x+1)(x-2)[x-(2-i)][x-(2+i)]$

$x^2-2x+x-2$ | $(x-2+i)(x-2-i)$

$x^2-2x-xi-2x+4-2i+xi-2i-i^2$

$x^2-2x-ix-2x+4+2i+ix-2i-(i)^2$

$x^2-4x+4+1 \rightarrow x^2-4x+5$

☆ Irreducible over the reals (Irreducible quadratic factors):
When a quadratic expression has no real zeros

x^2-8 ✓ $\sqrt{8} = \sqrt{4}\sqrt{2}$ $= 2\sqrt{2}$

$(x-2\sqrt{2})(x+2\sqrt{2})$

↳ Reduce Radicals!

$x^2+4 \rightarrow$ I.r., doesn't touch x-axis

Ex 7: ⓐ

$$\begin{array}{r|rrrrrr} 4 & 1 & 1 & -13 & -23 & -14 & -24 \\ & \downarrow & 4 & 20 & 28 & 20 & 24 \\ \hline & 1 & 5 & 7 & 5 & 6 & \boxed{R\ 0} ✓ \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & 5 & 7 & 5 & 6 \\ & \downarrow & -2 & -6 & -2 & -6 \\ \hline -3 & 1 & 3 & 1 & 3 & \boxed{R\ 0} ✓ \\ & \downarrow & -3 & 0 & -3 & \\ \hline & 1 & 0 & 1 & \boxed{R\ 0} ✓ & \end{array}$$

x^2+0+1

$(x-4)(x+2)(x+3)(x^2+1)$

$(x-4)(x+2)(x+3)(x-\sqrt{1})(x+\sqrt{1})$

ⓑ $(x-4)(x+2)(x+3)(x^2+1) \rightarrow x^2+1=0$

$\leftarrow \sqrt{x^2}=\sqrt{-1}$

$(x-4)(x+2)(x+3)(x-i)(x+i)$ $x=\pm i$

ⓒ $x=4,-2,-3,\pm i$

$\rightarrow$ 4 zeros, $2+5i$ & $2-5i$ are zeros

!★! Ex 8: $p(x)=x^{\textcircled{4}}-6x^3+35x^2-50x-58$ $\leftarrow$ Find all complex (just all) zeros

$x=2+5i$ is a zero of "p"

$$\begin{array}{r|ccccl} 2+5i & 1 & -6 & 35 & -50 & -58 \\ & \downarrow & 2+5i & -33-10i & 54-10i & 58 \\ \hline & 1 & -4+5i & 2-10i & 4-10i & \boxed{R\,0} \end{array}$$

$(2+5i)(-4+5i)$

$-8+10i-20i+25i^2$

$-8-10i-25 \rightarrow -33-10i$

$(2+5i)(2-10i)$

$4-20i+10i+50$

$54-10i$

$(2+5i)(4-10i)$

$8-20i+20i+50$

58

$$\begin{array}{r|cccl} 2-5i & 1 & -4+5i & 2-10i & 4-10i \\ & & 2-5i & -4+10i & -4+10i \\ \hline & 1 & -2 & -2 & \boxed{R\,0}\checkmark \end{array}$$

x^2-2x-2

$$x=\frac{+2\pm\sqrt{(-2)^2-4(1)(-2)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4+8}}{2}=\frac{2\pm\sqrt{12}}{2}\leftarrow \overset{4=\textcircled{2}\textcircled{2}}{\textcircled{3}}$$

$$=\frac{2\pm2\sqrt{3}}{2}$$

$[x-(2+5i)][x-(2-5i)][x-(1+\sqrt{3})][x-(1-\sqrt{3})]$ $\boxed{x=1\pm\sqrt{3}}$

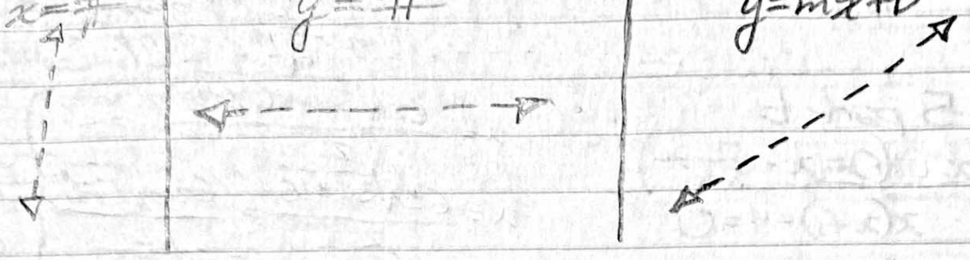
Oct. 9. 2023

2.5 Rational Functions

→ quotient of 2 polynomials ex: $\frac{a(x)}{b(x)}$; $b(x) \neq 0$

Asymptote → line that graph approaches, but never fully reaches

vertical	horizontal	oblique
$x = \#$	$y = \#$	$y = mx + b$



Ex 1:
$f(x) = \frac{2x^2+2x}{x^2+4x+3}$

① $\frac{(2x)(x+1)}{(x+3)(x+1)}$ ② $\{x \mid x \neq \overset{VA}{-3}, \overset{Hole}{-1}; x \in \mathbb{R}\}$
or $(-\infty,-3) \cup (-3,-1) \cup (-1,\infty)$

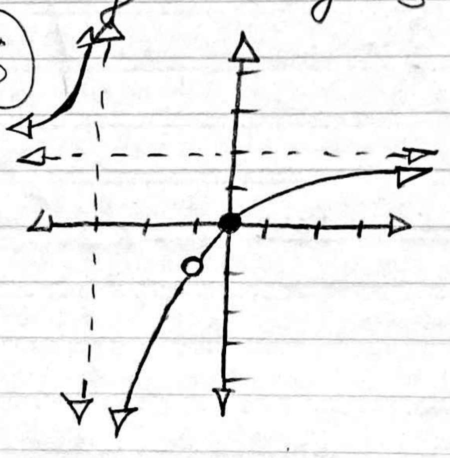
③ $\frac{2x}{x+3}$; Hole $\underline{\underline{(-1,-1)}}$ $\rightarrow \frac{2(-1)}{-1+3} \rightarrow \frac{-2}{2} \rightarrow -1$

④ VA: $x = -3$ ⑤ $y = \frac{2}{1}$; $y = 2$ ~~⑥~~ (no oblique if HA)

⑦ x-int
$y = \frac{2x}{x+3} \rightarrow x = 0 \rightarrow (0,0)$
$0 = 2x$

y-int
$y = \frac{2(0)}{0+3} \rightarrow y = \frac{0}{3} \rightarrow y \rightarrow (0,0)$

⑧



End behavior:

$\lim_{x \to \infty} f(x) = 2$ $\lim_{x \to -\infty} f(x) = 2$

VA behavior:

$\lim_{x \to (-3)^-} f(x) = \infty$

$\lim_{x \to (-3)^+} f(x) = -\infty$

$\mathcal{E}x\ 4: f(x) = \frac{x^2+x-8}{x+3}$

step

~~⑤~~ ⑥ $\begin{array}{r|rrr} \underline{-3} & 1 & 1 & \cancel{-8} \\ & \downarrow & -3 & \\ \hline & 1 & -2 & \end{array}$ ← Ignore Remainder

OA: $y = \underset{\frac{\text{rise}}{\text{run}}}{x} \underset{\text{"begin" y-int}}{-2}$

- - - - - - - - - - - - - - - -

2.5 part b

$\mathcal{E}x\ 6: \left(0 = x - \frac{4}{x-6}\right)^{x-6}$

$x(x-6) - 4 = 0$

$x^2 - 6x - 4 = 0$

$x = \frac{6 \pm \sqrt{36-4(1)(-4)}}{2}$

$x = \frac{6 \pm \sqrt{36+16}}{2} = \frac{6 \pm \sqrt{52}}{2}$

$= \frac{6 \pm 2\sqrt{13}}{2} = \boxed{3 \pm \sqrt{13}}$

} Look for domain

$\mathcal{E}x: \left(\frac{x}{x+2} + \frac{6}{x-5} = \frac{14}{\underset{(x-5)(x+3)}{x^2-3x-10}}\right)(x-5)(x+2)$

2.6 Non-linear Inequalites

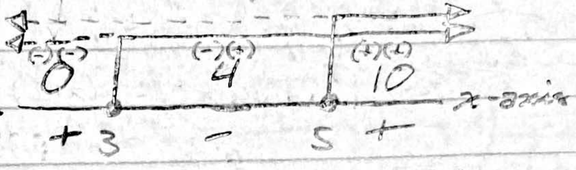
Polynomial Inequality: Poly.-func. with $>, <, \geq, \leq$ or $\neq 0$.

finding zeros
- ★factoring
- ★quad. form.
- ★P/Q

Ex 1 $x^2-8x+16 \leq 1$ ← Chart the signs } Solve

$\rightarrow x^2-8x+15 \leq 0$

$(x-5)(x-3) \leq 0$



$[3,5] \leq 0$

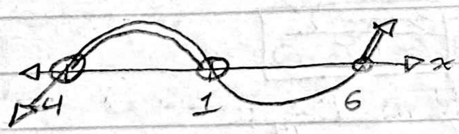
↳ shade "+" with —, "−" with ----

↳ Mutiply intervals

↓ finding zeros

Ex 2 $x^3-22x > 3x^2-24$ ← Use EB

$\underbrace{x^3-3x^2-22x+24}_{\text{"}y=\text{" see zeros}} > 0$



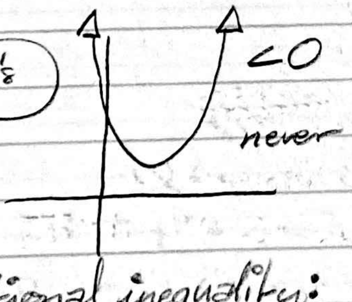
$(-4,1) \cup (6,\infty)$

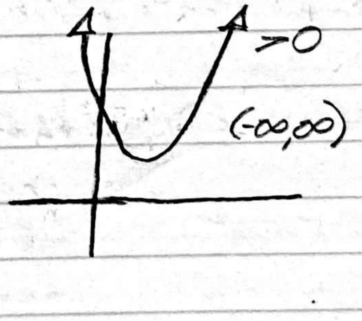
$x=-4$ cross

$x=1$ cross

$x=6$ cross

Ex's





Rational inequality:

1. Create common denominator
2. Put all fractions on one side of inequality → move to left side
   ↳ goal: $\frac{n}{d} \underset{\text{or } <}{>} 0$
3. Determine criticals
   - ↳ [zeros, set num. = 0]
   - ↳ (Points of discon. set denom. = 0) → Cannot be defined
4. Chart your signs or test intervals

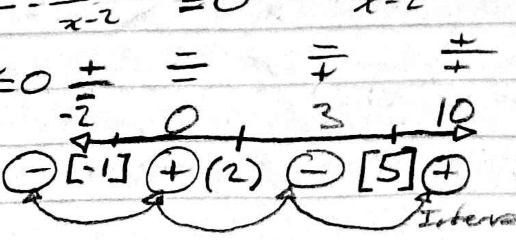
Ex 4 $\frac{x^2-x-11}{x-2} \leq 3$

$\frac{x^2-x-11}{x-2} - \frac{3(x-2)}{x-2} \leq 0$ $\rightarrow \frac{x^2-x-11}{x-2} - \frac{3x-6}{x-2} \rightarrow \frac{x^2-4x+5}{x-2} \leq 0$

zeros

Discon.

$\frac{(x-5)(x+1)}{x-2} \leq 0$



$(-\infty,-1] \cup (2,5]$

3.1 Exponential Functions

25.Oct.2023

Algebraic funcs.: uses variables & constants, and $+, -, \times, \div, \sqrt{n}, n^x$

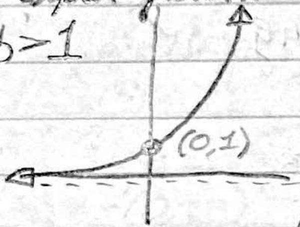
Transcendental funcs.: Exponential & Logarithmic funcs. Cannot be expressed in algebraic operations.

Transformations of Exponential Functions

Standard form: $f(x) = ab^{c(x-h)} + k$

Expo. Growth

$b > 1$



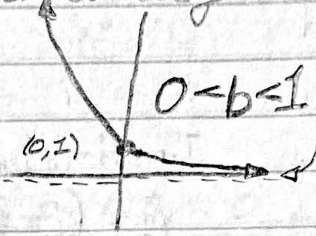
Parent Func. ↓

x	y
-1	$\frac{1}{b}$
0	1
1	b

$\boxed{b^x}$ ↑

Expo. Decay

$0 < b < 1$



Don't forget horizontal asymtote

Ex 1 B: $f(x) = \overset{\text{"b"}}{\left(\frac{1}{4}\right)}^x$; $a=1$, $k=0$, $c=1$

x	y
-1	$\frac{1}{0.25} = 4$
0	1
1	0.25

Domain: $(-\infty, \infty)$

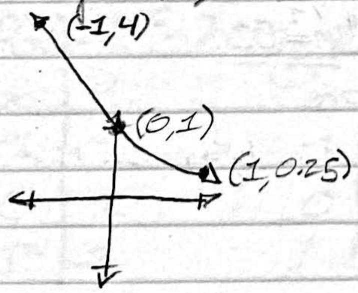
Range: $(0, \infty)$

x-int: none

y-int: $y=1$

asy: $y=0$

EB: $\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to-\infty} f(x) = \infty$



inc: never dec: $(-\infty, \infty)$

Tables & Transformations:

1) Create parent func. table
2) Factor out "c"
3) Multiply by dialations/reflections
   - a: multi. to y-values
   - $\frac{1}{c}$: multi. to x-values
   - use reciprocal of "c"!
4) add translations
   - h: add to all x-values
   - k: add to all y-values

Ex 2:B

$f(x)=5^x$

$g(x)=-(5)^{2x+6}-1$

① 5^x ② $-(5)^{2(x+3)}-1$ ③ ④ Second func.

x	y
$-1^{(\frac{1}{2})}$	$\frac{1}{5}(-1)$
$0^{(\frac{1}{2})}$	$1(-1)$
$1^{(\frac{1}{2})}$	$5(-1)$

Initial func.

a+c

$a:-1$

$c:2 \rightarrow (\frac{1}{2})$ recip.

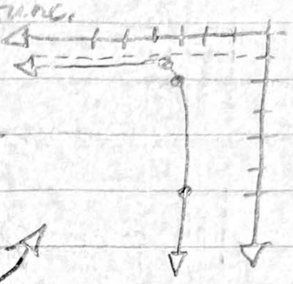
$h:-3$

$k:-1$ ↳ HA

x	y
$-\frac{1}{2}^{-3}$	$-\frac{1}{5}^{-1}$
0^{-3}	-1^{-1}
$\frac{1}{2}^{-3}$	-5^{-1}

h+k →

x	y
$-3\frac{1}{2}$	$-1\frac{1}{5}$
-3	-2
$-2\frac{1}{2}$	-6



4 graph

Natural Base: Irrational number represented by:

$e=2.718281828\ldots$ $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Compound Interest: $A(t)=P\left(1+\frac{r}{n}\right)^{nt}$

A: Amount after "t" years

P: Principal

r: Annual interest rate → change to decimal

n: Number of times compounded → "weekly" $\underline{\underline{n=52}}$

$\underline{t: \text{years}}$ → convert all other forms to years

Continuously Compounded Interest:

$A(t)=Pe^{rt}$

Ex 4 A:

$A(18)=2000\left(1+\frac{.05}{4}\right)^{4(18)}$ → 4891.84$

B: = 4910.02 $

C: = 4918.90$

Ex 5 = 4919.21$

Exponential Growth/Decay:

$N(t)=N_0(1\pm r)^t$ → N_0 = Amount at start time = P

Continuous Exponential Growth/Decay:

$N(t)=N_0e^{\pm kt}$

3.2 Logarithmic Functions

Oct. 27.2023

Log func.: $\overset{y}{f(x)}=\log_b x$ where $b>0$, $b\neq 1$, $x>0$

↳ Inverses: $\log_b x$ and b^x

Relating Log and Exponential Forms:

If $b>0, b\neq 1$ and $x>0$, then

Log form: $\log_{\textcircled{b}} x = y$ if & only if Expo. form: $\textcircled{b}^{y}=x$

convert: $b^y=x$

convert: $\log_b x = y$

Ex 1

Ⓐ $\log_{\textcircled{2}} 16 = x$

$2^x=16$

$\boxed{x=4}$

Ⓑ $\log_{\textcircled{5}} \frac{1}{125} = x$

$\frac{1}{125}=\frac{1}{5^3}=5^{-3}$

$5^x=5^{-3}$

$\boxed{x=-3}$

Ⓒ $\log_{\textcircled{3}} \frac{1}{27} = x$

$\frac{1}{3^3}=3^{-3}$

$3^x=3^{-3}$

$\boxed{x=-3}$

ⓓ

Properties of Logs.:

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$
- $b^{\log_b x} = x, x>0$

} same for common & natural logs

Ex 2

Ⓐ $\log_8 512$

$\cancel{\log_8} 8^3$

$\boxed{=3}$

512 → ⑧, 64

64 → ⑧⑧

Ⓑ $22^{\log_{22} 15.2} = \boxed{15.2}$

Common Log

Log with base 10

- $\log 1 = 0$
- $\log 10 = 1$
- $\log 10^x = x$
- $10^{\log x} = x, x>0$

Ex 3

Ⓐ $\log_{\textcircled{10}} 10{,}000 = x$

$10^x = 10{,}000$

$\boxed{10^4}$

⑩ 1000

⑩ 100 = ⑩⑩

Ⓒ $\log 14$

↳ 1.146

ⓓ $\log -11$

↳ $x<0$

↳ Undefined

Natural Logs: log with base e → $\log_e = \ln$

Properties:

$\ln 1 = 0$

$\ln e = 1$

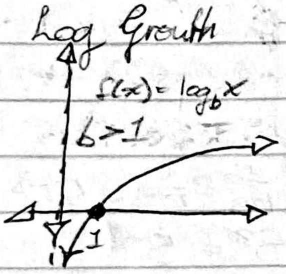
$\ln e^x = x$

$e^{\ln x} = x, x > 0$

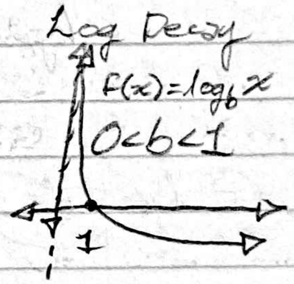
(Ex4) Ⓐ $\ln e^{4.6} = 4.6$

Ⓓ $\ln 7 = 1.95$

Log Growth



Log Decay



x	y
$\frac{1}{b}$	-1
1	0
b	1

← Inverse of expo. table!

↔

x	y
-1	$\frac{1}{b}$
0	1
1	b

(Ex 5)

$\to -1 \quad \to -\frac{1}{2} \quad \to \boxed{-4} \to VA \quad \to 3$

Ⓐ $-\log(-2x-8)+3 \to -\log -2(x+4)+3$ Domain: $(-\infty, -4)$ Range: $(-\infty, \infty)$

①

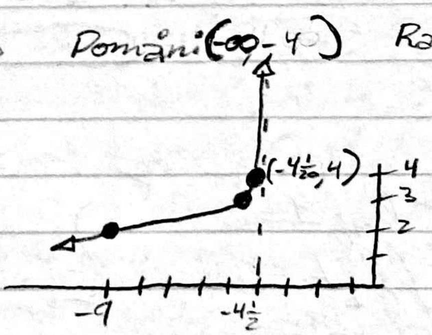
x	y
$\frac{1}{10}$	-1
1	0
10	1

②

x	y
$-\frac{1}{20}$	1
$-\frac{1}{2}$	0
-5	-1

③

x	y
$-4\frac{1}{20}$	4
$-4\frac{1}{2}$	3
-9	2



x-int: ~-x   y-int: none

VA: $x = -4$ $\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to -4^-} f(x) = \infty$ inc.: $(-\infty, -4)$

to change base, divide by desired base:

$$\log_8 x \to \frac{\log x}{\log 8}$$

$\log_4 5 = \frac{\log 5}{\log 4}$

3.3 Properties of Logs

Oct. 30. 2023

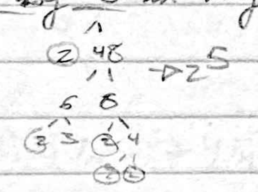
↳ Product : $\log_b xy = \log_b x + \log_b y$

Quotient : $\log_b \frac{x}{y} = \log_b x - \log_b y$

Power : $\log_b x^p = p \log_b x$

Ex 1 Ⓐ

$\log 96$ in $\log 2$ & $\log 3$



$\log 96 = \log(2^5 \cdot 3 \rightarrow 5\log 2 + \log 3$

$\log(2^5) + \log 3$

↑ Not done!

Ⓑ $\log \frac{32}{9}$ in $\log 2$ & $\log 3$

$\log 32 - \log 9 \rightarrow \log 2^5 - \log 3^2$

$5 \log 2 - 2 \log 3$

$\sqrt[3]{2^1} \rightarrow 2^{1/3}$

Ex 2 Ⓐ

$\log_2 \sqrt[3]{32} = x \rightarrow 2^x = \sqrt[3]{32} \rightarrow 2^x = \sqrt[3]{2^5} \rightarrow x = 5/3$

$2^x = 3.1748...$ $\cancel{2}^x = \cancel{2}^{5/3}$

Ⓑ $3 \ln e^4 - 2 \ln e^2 \rightarrow$

$3(4) - 2(2)$ $12 - 4 \rightarrow x = 8$

Ex 3 Ⓐ

$\ln(4m^3n^5) \rightarrow \ln 4\overset{\ln}{m}^3 + \ln n^5 \rightarrow$

$\ln 4 + 3 \ln m + 5 \ln n$

Ⓑ $\log \frac{2x-3}{3\sqrt[4]{x}} \rightarrow \log(2x-3) - \log(3\sqrt[4]{x}) \rightarrow \log(2x-3) - \log(3(x^{1/4}))$

$\log(2x-3) - \log 3 - \log x^{1/4} \rightarrow \log(2x-3) - \log 3 - (\frac{1}{4}) \log x$

Ex 4 Ⓐ

$\left(\frac{1}{2}\right)\log_4 x - 3\log_4(x-2) \rightarrow \log_4 x^{1/2} - \log_4(x-2)^3$

$\log_4\left(\frac{x^{\frac{1}{2}}}{(x-2)^3}\right) \rightarrow \log_4\left(\frac{\sqrt{x}}{(x-2)^3}\right)$

Ⓑ $5\ln(x+1) + 6\ln x \rightarrow \ln[(x+1)^5(x^6)]$

$\ln(x+1)^5 + \ln x^6$

Change of Base Formula

$$\log_b x = \frac{\log x}{\log b}$$

&*

3.4 Expo. & Log. Equations

Nov. 1. 2023

$\boxed{b^x = b^y}$ if & ONLY IF $\boxed{x=y}$ ← Exponential

One-to-One Properties

$\boxed{\log_b x = \log_b y}$ if & ONLY IF $\boxed{x=y}$ ← Logarithmic

Ex 1 Ⓐ

$4^{x+2} = 16^{x-3}$ ← common base?

$4^{x+2} = (4^2)^{x-3} \rightarrow 4^{x+2} = 4^{2x-6} \rightarrow \underset{-x+6}{x+2} = \underset{-x\ +6}{2x-6}$

$\rightarrow \boxed{x = 8}$

Ⓑ $\left(\frac{1}{3}\right)^n = \left(\frac{1}{81}\right)^{\frac{2}{3}} \rightarrow (3^{-1})^n = (81^{\cancel{-1}})^{\frac{2}{3}(\frac{1}{1})} \rightarrow 3^{-n} = 81^{-\frac{2}{3}} = 3^{4(-\frac{2}{3})}$

$3^{-\frac{8}{3}}$

$-n = -\frac{8}{3} \rightarrow \boxed{n = \frac{8}{3}}$

Solving Log Equations:

Type 1

$\log_{b} x = \#$ Convert to Expo form, solve, check extraneous solutions

Type 2

$\log_b x = \log_b y$ } 1-to-1

$\rightarrow x = y$

Solve, check for extraneous solutions!

Ex 2 Ⓐ

$2\ln x = 18 \rightarrow \ln_e x = 9 \rightarrow e^9 = x \rightarrow \boxed{8103.0839}$

$\rightarrow > 0$ ✓

Ⓑ $7 - 3\log(10x) = 13$

$-3\log(10x) = 6$

$\log_{10}(10x) = -2$

$10^{-2} = 10x$

$\frac{0.01}{10} = \frac{10x}{10} \rightarrow x = 0.001$ or $\frac{1}{1000}$

$\boxed{x \approx 0}$ $\rightarrow > 0$ ✓

$10^{-2} \rightarrow \frac{1}{10^2} \rightarrow \frac{\frac{1}{100}}{10} \rightarrow \frac{1}{100} \cdot \frac{1}{10} \rightarrow \frac{1}{1000}$

Ex 3 Ⓐ

$\log_2 5=\log_2 10-\log_2 (x-4)$

$\log_2 5=\log_2 \frac{10}{x-4} \rightarrow \underset{\cdot(x-4)}{5}=\frac{10}{x-4}\cdot x-4$

$5x-20=10 \rightarrow 5x=30 \rightarrow \boxed{x=6}>0$ ✓

$\hookrightarrow x-4 \rightarrow 6-4=2>0$ ✓

Ⓑ

$\log_5(x^2+x)=\log_5 20 \rightarrow x^2+x=20 \rightarrow x^2+x-20=0$

$(x+5)(x-4)=0$

$20,\ 25-5,\ 16+4 \} > 0$ ✓ $\boxed{x=-5, 4}$

Ex 4 Ⓐ ⌐ No common bases!

meant to write $\log_{10}$!

$3^x=7$

$\hookrightarrow \ln 3^x=\ln 7 \rightarrow \frac{x \ln 3}{\ln 3}=\frac{\ln 7}{\ln 3} \rightarrow x=\frac{\ln 7}{\ln 3}=1.77$

$\hookrightarrow =\log_3 7$

✱ Ⓑ $e^{2x+1}=8$ | $2x=\ln(8)-1$ $\rightarrow \boxed{x=0.5397}$ >0 ✓

$\ln e^{2x+1}=\ln 8$ | $x=\frac{\ln(8)-1}{2}$ ← Exact ↑ Numerical Answers

$2x+1=\ln 8$ |

Ex 5

$3^{6x-3}=2^{4-4x} \rightarrow \log 3^{6x-3}=\log 2^{4-4x}$

$(6x-3)\log 3=(4-4x)\log 2$

$6x\log 3-3\log 3=4\log 2-4x\log 2$

$+4x\log 2 \quad +3\log 3 \qquad +4x\log 2 \quad +3\log 3$

$6x\log 3+4x\log 2=4\log 2+3\log 3$

$x(\underbrace{6\log 3+4\log 2}_{(11)\leftarrow})=\frac{(4\log(2)+3\log(3))}{(6\log(3)+4\log(2))} \rightarrow \boxed{x=0.648}$

3.4 part 2

Ex 6

$(1)e^{2x}(1)e^{x}-2=0$

$(e^{x})^{2}-(e^{x})-2=0$

$u^{2}-u-2=0$

$(u-2)(u+1)=0$

$u=-1,2$

$e^{x}=-1$

$\ln e^{x}=\ln(-1)$

$x=\ln -1$

$x=$ undefined

$e^{x}=2$

$\ln e^{x}=\ln 2$

$x=\ln 2$ ← exact form

$\boxed{x=0.693}$ ← numerical

Ex 8

$\log(3x-4)=1+\log(2x+3) \rightarrow \log(3x-4)-\log(2x+3)=1$

$1=\log_{10}\frac{3x-4}{2x+3} \rightarrow 10^{1}=\frac{3x-4}{2x+3}$

3.5 $\boxed{\text{WU}}$ Nov. 3, 2023

① $9^{x-2}=27^{3x} \rightarrow (3^2)^{x-2}=(3^3)^{3x}$

$\rightarrow 2x-4=9x \rightarrow -4=7x \rightarrow x=-\frac{4}{7}$

② $3+5\log(2x)=8 \rightarrow 5\log(2x)=5$

$\log_{10}(2x)=1 \rightarrow 10=2x \rightarrow x=5$

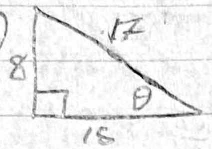
③ $\log_5(3x)=\log_5 2+\log_5(x+2)$

$\log_5(3x)=\log_5(2(x+2) \rightarrow 3x=2x+4 \rightarrow x=4$

④ $3e^{5x}=74 \rightarrow e^{5x}=\frac{74}{3} \rightarrow 5x=\ln\frac{74}{3} \rightarrow x=\frac{(\ln\frac{74}{3})}{5}$

$x\approx 0.641$

4.2 Warmup

① 

$64+15^2=c^2$ $\sin\theta=\frac{8}{17}$ $\cos\theta=\frac{15}{17}$ $\tan\theta=\frac{8}{15}$

$\csc\theta=\frac{17}{8}$ $\sec\theta=\frac{15}{17}$ $\cot\theta=\frac{15}{8}$

$9+b^2=25$

② $\cos\theta=\frac{3}{5}$ $\sec\theta=\frac{5}{3}$ $\sin\theta=\frac{4}{5}$ $\tan\theta=\frac{4}{3}$

adj=3 o=4 $\csc\theta=\frac{5}{4}$ $\cot\theta=\frac{3}{4}$

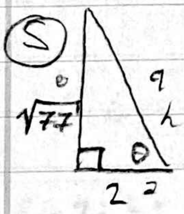
H=5

③ $\cos 59=\frac{x}{23}\to 23\cos 59=x\to x=11.8$ c

$23\sin 59=x\to x=19.7$a $c=31$

④ $\cos 61=\frac{x}{18}\to 18\cos 61=x\to x=8.7$

toa

⑤ 

$4+b^2=81$ $b^2=77$

$\tan\theta=\frac{\sqrt{77}}{2}$

$\sin^2x+\cos x\sin x+\cos x\sin x+\cos^2x$

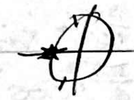
$(\sin x+\cos x)(\sin x+\cos x)$

5.4 Warm up

ⓐ $2\cos^2x+3\cos x+1=0\to\cos^2x+3\cos x+2$

$(\cos x+2\quad)(\cos x+1\quad)\to(\cos x+1)(2\cos x+1)$

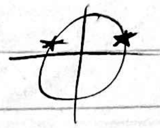
$\cos x=-1,-\frac{1}{2}$ d?



$\pi,\frac{2\pi}{3},\frac{4\pi}{3}$

ⓑ $(\sin x+\cos x)^2=(1)^2\to\sin^2x+2\sin x\cos x+\cos^2x=1$

ⓒ $4\sin\theta-1=2\sin\theta\to 2\sin\theta-1=0\to\sin\theta=\frac{1}{2}$



$\frac{\pi}{6},\frac{5\pi}{6}$

$\sin x+\cos x=1$, $[0,2\pi)$

$(\sin x+\cos x)^2=1^2$

$\to(\sin x+\cos x)(\sin x+\cos x)$

$\to\sin^2x+\sin\cos+\sin\cos+\cos^2x=1$

$\sin^2x+2\sin x\cos x+\cos^2x=1$

$1+2\sin x\cos x=1$

$\to 2\sin x\cos x=0$

$\to\sin x\cos x=0$

$\sin\pi+\cos\pi=1$

$0-1=1$

$\sin x=0$

$\cos x=0$

$\sin 0+\cos 0=1$

$\to 0+1=1$

$\sin\frac{\pi}{2}+\cos\frac{\pi}{2}=1$

$1+0=1$

$\sin\frac{3\pi}{2}+\cos\frac{3\pi}{2}$

$=$

1

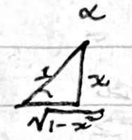
$\to 0-1=1$

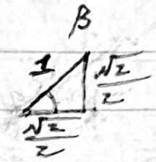
$x=\textcircled{0}\ \textcircled{\frac{\pi}{2}}$ [illegible]

5.5 Warm-up

ⓐ $\tan\frac{\pi}{12}$ <u>a</u>

ⓑ $\frac{5\pi}{12} \to \frac{2\pi}{12} + \frac{3\pi}{12} \to \boxed{\frac{\pi}{6} + \frac{\pi}{4}}$





ⓒ

ⓓ $\sin\frac{x}{1} \cdot \cos\frac{\sqrt{2}}{2} + \sin\frac{\pi}{4} \cdot \cos\frac{\sqrt{1-x^2}}{1} + \sin\frac{x}{1} \cdot \cos\frac{\sqrt{2}}{2} - \sin\frac{\sqrt{2}}{2} \cdot \cos\frac{\sqrt{1-x^2}}{1}$

$\frac{\pi}{4}$

ⓐ $\tan\frac{\pi}{12} \to$

$\cos\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$, $\sin\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$

ⓑ $\sin\frac{5\pi}{12}\cos\frac{\pi}{4} - \cos\frac{5\pi}{12}\sin\frac{\pi}{4} \to$

½ $\sin\left(\frac{5\pi}{12} - \frac{3\pi}{12}\right) \to \sin\frac{2\pi}{12} \to \sin\frac{\pi}{12}$

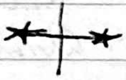
<u>a caa</u>

$\to \frac{10\pi}{24} \to \frac{2.5\pi}{6}$

ⓓ $\sin x \cdot \cos\frac{\pi}{4} + \sin\frac{\pi}{4} \cdot \cos x + \sin x \cdot \cos\frac{\pi}{4} - \sin\frac{\pi}{4} \cdot \cos x$

$\to \sin x \frac{\sqrt{2}}{2} + \cancel{\cos x \frac{\sqrt{2}}{2}} + \sin x \frac{\sqrt{2}}{2} - \cancel{\cos x \frac{\sqrt{2}}{2}}$

$2 \sin x \frac{\sqrt{2}}{2} = 0 \to \sin x \sqrt{2} = 0 \to \sin x = 0$



ⓑ $\sin\frac{5\pi}{12} \to \sin\frac{2\pi}{12} + \frac{3\pi}{12} \to \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$\sin\frac{\pi}{6} \cdot \cos\frac{\pi}{4} + \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{6} \to \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \to \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\to \frac{\sqrt{2}+\sqrt{6}}{4} \cdot \frac{\sqrt{2}}{2} \to \frac{\sqrt{4}+\sqrt{12}}{8} -$

$\cos\frac{\pi}{6} + \frac{3\pi}{4} \to \cos\frac{\pi}{6} \cdot \cos\frac{\pi}{4} - \sin\frac{\pi}{6} \cdot \sin\frac{\pi}{4} \to \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

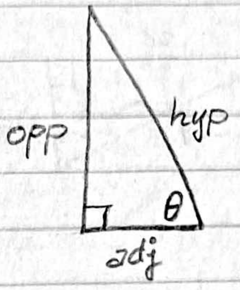
$\to \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \to \frac{\sqrt{6}-\sqrt{2}}{4}$

$\frac{\sqrt{4}+\sqrt{12}}{8} - \frac{\sqrt{6}-\sqrt{2}}{4} \cdot \frac{\sqrt{2}}{2} \to \frac{\sqrt{4}+\sqrt{12}}{8} - \frac{\sqrt{12}-\sqrt{4}}{8} \to \frac{2\sqrt{4}}{8} \to \frac{4}{8} \to \boxed{\frac{1}{2}}$

$$\frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

4.1 Right Triangle Trig.

Nov. 15.2023



θ= variable angle

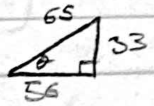
Sine	SOH	$\frac{opp}{hyp}$	$\sin \theta$
Cosine	CAH	$\frac{adj}{hyp}$	$\cos \theta$
Tangent	TOA	$\frac{opp}{adj}$	$\tan \theta$

Cosecant $\frac{1}{\sin\theta} = \frac{hyp}{opp}$ $\csc \theta$

Secant $\frac{1}{\cos\theta} = \frac{hyp}{adj}$ $\sec \theta$

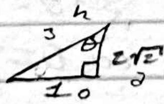
Cotangent $\frac{1}{\tan\theta} = \frac{adj}{opp}$ $\cot \theta$

Ex:



$a^2 + 56^2 = 65^2$ $\sqrt{a^2} = \sqrt{1089}$ $a = 33$

$\sin\theta = \frac{33}{65}$



$a^2 + 1^2 = 3^2$ $a^2 = 8$ $a = \sqrt{8}$

$a = 2\sqrt{2}$

Ex 2: $\sin\theta = \frac{1}{3}\frac{opp}{hyp}$

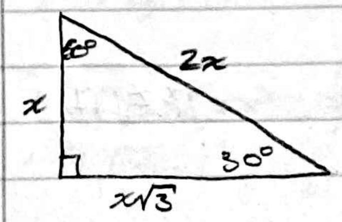
$\cos\theta = \frac{2\sqrt{2}}{3}$

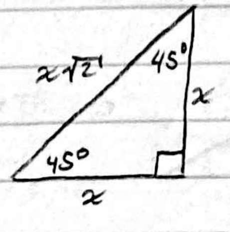
$\tan\theta = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{4}$

Special Right Triangles:

30°-60°-90° Triangle

45°-45°-90° Triangle

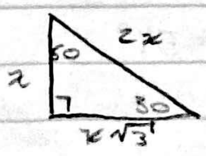


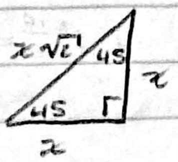


ex: csc 60° $\frac{2x}{x\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc\theta = \frac{hyp}{opp}$

ex: tan 45° $\frac{x}{x} = 1$





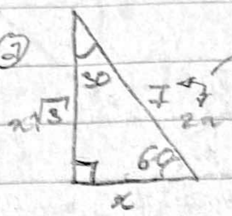
$x\sqrt{3}$ $x\sqrt{2}$
$x\sqrt{3}$ $x\sqrt{2}$
$x\sqrt{3}$ $x\sqrt{2}$

When "exact value" used in instructions, don't use calculator

4.1 part b

Nov. 27. 2025

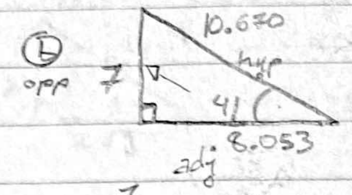
Ex 3: ⓐ



$7 = 2x \rightarrow x = \frac{7}{2}$ $\quad x\sqrt{3} = \frac{7\sqrt{3}}{2}$

SOH CAH TOA

ⓑ

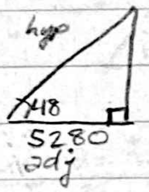


$\sin 41 = \frac{7}{hyp} \rightarrow \frac{hyp \sin 41 = 7}{\sin 41 \quad = \sin 41}$

$hyp = \frac{7}{\sin 41} \rightarrow hyp = 10.670$ $\quad$ Toa $\rightarrow \tan 41 = \frac{7}{x} = x\tan 41 = 7$

$adj = 7/\tan 41 = 8.053$

Ex4:



$\cos 48 = \frac{5280}{x} \rightarrow x\cos 48 = 5280$

$hyp = \frac{5280}{\cos 48} \rightarrow hyp = 7890$ ft.

Inverse Trig functions used to find angle measure!

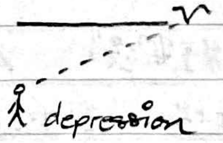
$\sin\theta = \frac{opp}{hyp} \rightarrow \sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{opp}{hyp}\right) \rightarrow \theta = \sin^{-1}\left(\frac{opp}{hyp}\right)$ ☆

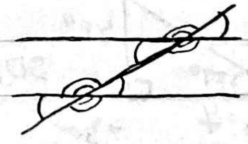
Different from "csc," "sec," and "cot"!

Ex 5: hyp = 15.7 $\quad$ opp = 12 $\quad \sin^{-1}\left(\frac{12}{15.7}\right) = \theta \rightarrow \theta = \sim 50°$

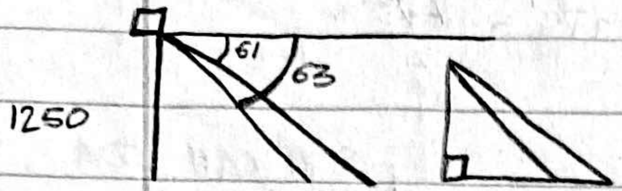
☆ ∡ of elevation + depression

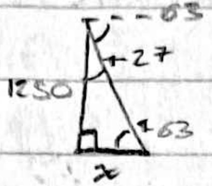
↳ ∡ formed by lign of sight + a horizontal line

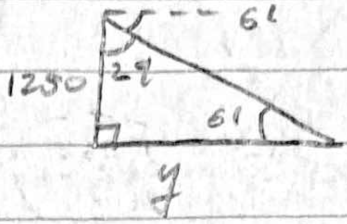




Ex 7:







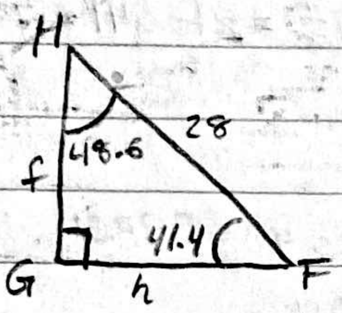
x $\tan 27 = \frac{x}{1250}$ $x = 1250 \tan 27$ $x = 636.907$

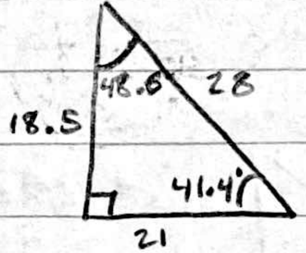
y $\tan 29 = \frac{y}{1250}$ $y = 1250 \tan 29$ $y = 692.886$

$y - x \approx \boxed{56 \text{ ft}}$

$\begin{array}{r} 90 \\ -41.4 \end{array}$ $\begin{array}{r} 49 \\ 48.6 \end{array}$

Ex 8:



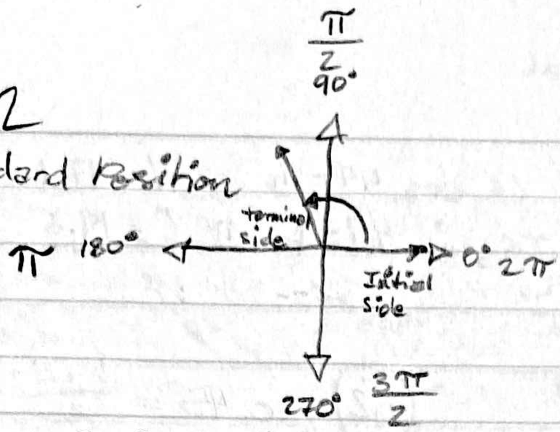


f $\sin 41.4 = \frac{f}{28} \rightarrow 28 \sin 41.4 = f \rightarrow f = 18.517$

h $\sin 48.6 = \frac{h}{28} \rightarrow 28 \sin 48.6 = h \rightarrow h = 21.003$

4.2

Standard Position



DMS $\overbrace{\text{Degree, Minutes}}^{[2ND]\quad[APPS]}$, Seconds

$1° = 60'$ (minutes) $1° = 3600''$ (seconds) $1' = 60''$

[Ex 1] ⓐ 329.125° → DMS → 329° 7' 30''
ⓑ 35° 12' 7'' → 35.202°

Degree $\leftrightarrow$ Radian conversion

$360° = 2\pi \longleftrightarrow 2\pi = 360°$ ★ $\frac{360°}{360°} = \frac{2\pi}{360} \to \boxed{1° = \frac{\pi}{180} r}$

$\hookrightarrow \frac{2\pi}{2\pi} = \frac{360°}{2\pi} \to \boxed{1r = \frac{180°}{\pi}}$

Ex 2: ⓐ 135° → r

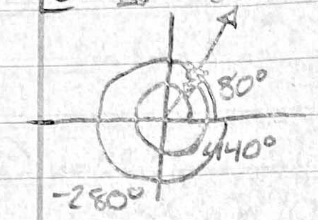
$$\frac{135}{1}\left(\frac{\pi}{180}\right) = \frac{135\pi}{180} \to \frac{27\pi}{36} \to \frac{9\pi}{12} \to \frac{3\pi}{4}$$

ⓒ $\frac{2\pi}{3}\left(\frac{180}{\pi}\right) = 120°$

Coterminal Angles: When 2+ ∠ have same initial + terminal sides, but different measures

Degree	Radian	
$\alpha + 360n$	$\alpha + 2\pi n$	$\twoheadleftarrow n \in \mathbb{Z}$
α: degrees	α: radians	
n: integer	n: integer	

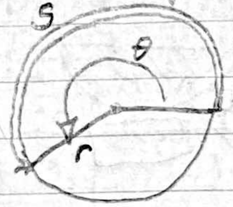
[Ex 3] $80° + 360n, n \in \mathbb{Z}$



$80+360=440°$ $80-360=-280°$

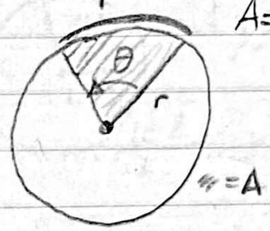
Arc Length: $s = r\theta$

s = arc length r = radius θ = central angle in radians

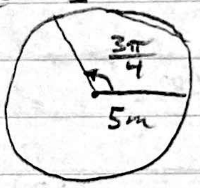


Area of Sector: $A = \frac{1}{2} r^2 \theta$

A = Area r = radius θ = central angle in radians



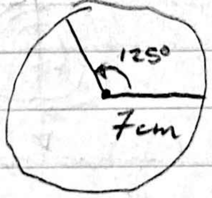
[Ex 6]



$A = \frac{1}{2} 5^2 \left(\frac{3\pi}{4}\right)$

$A = 29.452 m^2$

[Ex 4] 125



$\frac{125}{1}\left(\frac{\pi}{180}\right)$

$\downarrow$

$s = 7(\ '' \ ) \approx 15.272 \text{ cm}$

★ Linear Speed: $v = \frac{s}{t}$

s: arc length t: time

v = Linear speed

Angular speed: $\omega = \frac{\theta}{t}$

ω = angular speed

θ = angle of rotation (radians)

t = time

Radian → Revolution Conversion: 1 revolution = 2π radians

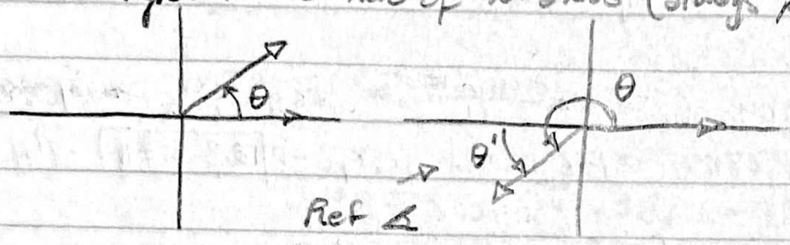
[Ex 5] $33\frac{1}{3}$ rev/min $\rightarrow \frac{33\frac{1}{3}\text{rev}}{1\text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{rev}} \rightarrow \frac{209.4395r}{1 \text{ min}}$

$\frac{\theta}{t}$

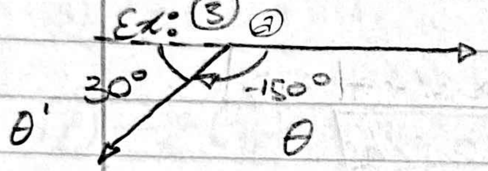
4.3 Trig func. on Unit Circle

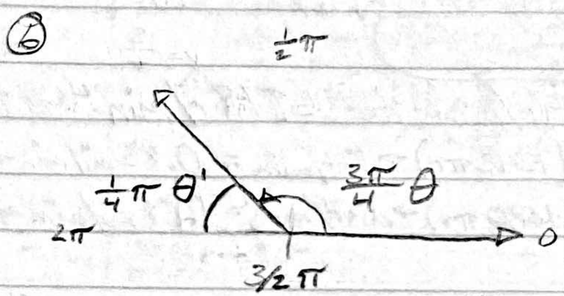
Dec.1.2023

Reference Angle: Terminal side of x-axis (always positive)

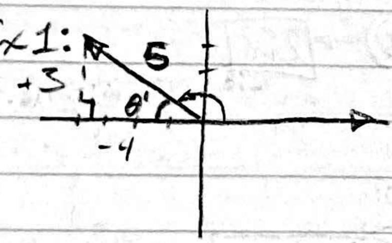


Ex: ③





Ex 1:



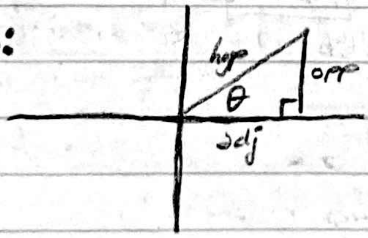
$3^2+4^2=c^2$

$9+16=c^2$

$25=c^2$

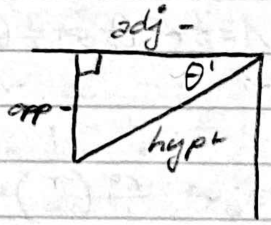
$\cot\theta' = \frac{-4}{3}$

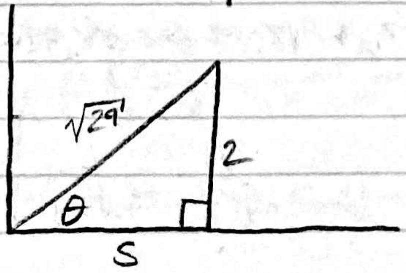
Ex 5:



$\sin\theta = \frac{+}{+}$

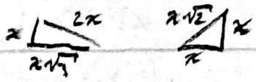
$\cos\theta = \frac{+}{+}$



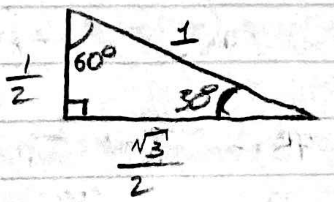


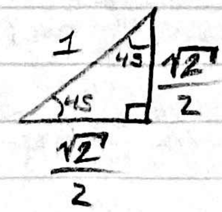
$25+b^2=29$ $b^2=4$ $b=2$

Rember!

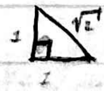


$1=x\sqrt{2}$ $\frac{1}{\sqrt{2}}=x \to \frac{\sqrt{2}}{2}$





sin 90



$\frac{1}{\sqrt{2}}$

4.4 Graphing Sine & Cosine

Dec. 4. 2023

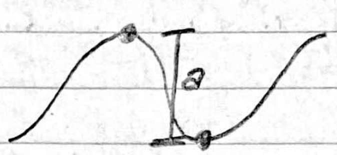
Sinusoid → Any transformation of a sine function

Standard form of sin & cos: $x = \theta$

$f(x) = a\sin(cx - h) + k$ $f(x) = a\cos(cx - h) + k$

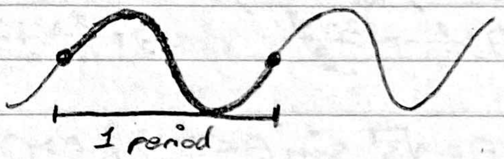
Amplitude: Half dist. between min + max values of sin func.

a: $a < 0$: Reflect over x-axis
$|a| > 1$: Vert. expand
$0 < |a| < 1$: Vert. compression



Period: Distance a func. takes to complete a cycle

c: $\dfrac{2\pi}{|c|}$ } Works with:
sin cos
csc sec



Phase Shift:

Horizontal translation → h-value

$a\sin(cx - h) + k$

$\to a\sin\left[c\left(\frac{x}{c} - \frac{h}{c}\right)\right] + k \to \dfrac{h}{|c|}$

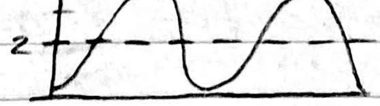
Frequency $= \frac{c}{2\pi}$

St	e	a	k
s t a r t	e n d	a	k

Midline:

Refrence line graph oscillates about

$y = k$



→ k is vert. translation

Ex 5: "h" is always negative!

$f(x) = 2\sin\left(2x + \frac{\pi}{4}\right) + 1$

$\dfrac{\frac{\pi}{4}}{2}$

$\frac{\pi}{4} \cdot \frac{1}{2} \to \frac{\pi}{8}$

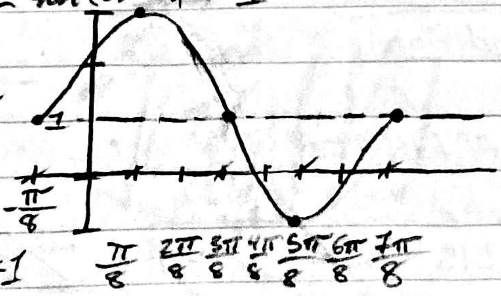
amp: 2

per: π

PS: $-\frac{\pi}{8}$

VS: 1

ME: $y = 1$

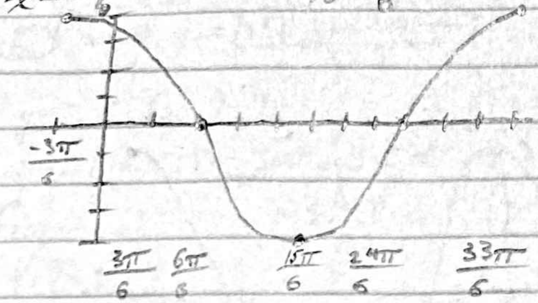


Start	End
$2x + \frac{\pi}{4} = 0$	$2x + \frac{\pi}{4} = 2\pi \to \frac{8\pi}{4}$
$2x = -\frac{\pi}{4}$	$2x = \frac{7\pi}{4}$
$x = -\frac{\pi}{8}$	$x = \frac{7\pi}{8}$

$\frac{\pi}{6} \cdot \frac{3}{1} \rightarrow \frac{3\pi}{6}$

$y=4\cos\left(\frac{x}{3}+\frac{\pi}{6}\right) \rightarrow y=4\cos\left[\frac{1}{3}\left(x+\frac{3\pi}{6}\right)\right]$

Start	End
$\frac{x}{3}+\frac{\pi}{6}=0$	$\frac{x}{3}+\frac{\pi}{6}=2\pi$
$\frac{x}{3}=-\frac{\pi}{6}$	$\frac{x}{3}=\frac{2\pi}{1} \rightarrow \frac{12\pi}{6} \rightarrow \boxed{\frac{11\pi}{6}}$
$x=-\frac{3\pi}{6}$	$x=\frac{33\pi}{6}$



4.5 Graphing Other Trig Functions

Jan. 5. 2024

standard form: $f(x) = a \tan(cx - h) + k$

period: $\frac{\pi}{|c|}$

Ex 1: $y = 2\csc(x + \frac{\pi}{3}) + 1$

Start

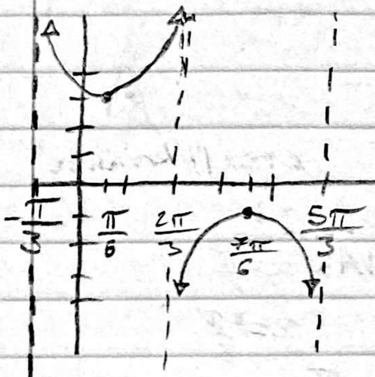
$x + \frac{\pi}{3} = 0$

$x = -\frac{\pi}{3}$

End

$x + \frac{\pi}{3} = 2\pi$

$\left.\begin{array}{l} x = \frac{2\pi}{1} - \frac{\pi}{3} \\ x = \frac{6\pi}{3} - \frac{\pi}{3} \end{array}\right\} x = \frac{5\pi}{3}$



$x = -\frac{\pi}{3}$

$x = \frac{2\pi}{3}$ $x = \frac{5\pi}{3}$

Inverse finds angle!

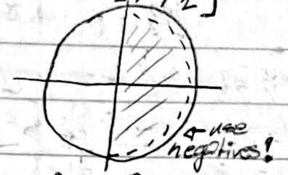
Jan 8
2024

4.6 Inverse Trig Funcs

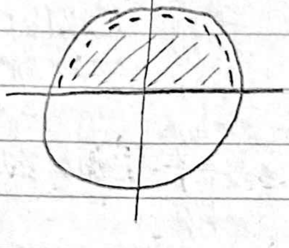
★ In order for a function to have an inverse, it must meet vertical line test

Inverse Trig Func.s

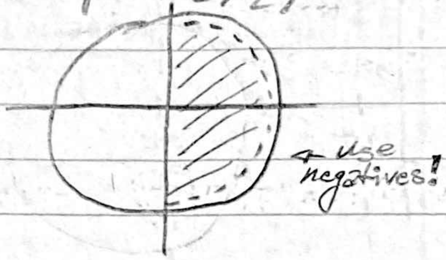
$\boxed{\sin^{-1}}$
Domain: $[-1, 1]$
Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$



$\boxed{\cos^{-1}}$
Dom: $[-1, 1]$
Range: $[0, \pi]$

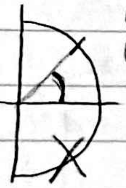


$\boxed{\tan^{-1}}$
Dom: $(-\infty, \infty)$
Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$



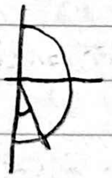
Arcsine $\theta = \sin^{-1}$

Ex 1: Ⓐ $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$



(cos, sin)
(x, y)
$45° \rightarrow \boxed{\frac{\pi}{4}}$

Ⓑ $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$



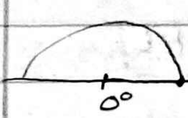
$\boxed{-\frac{\pi}{3}}$

Ⓒ $\sin^{-1}(-2\pi)$

undefined! ↳ -6.28...
↳ $[-1, 1]$

Cosine^{-1} = Arccosine θ

Ex 2: Ⓐ $\cos^{-1} 1$



$\boxed{0\pi r}$

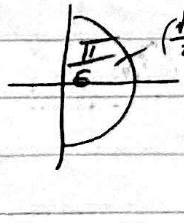
Ⓑ $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{5\pi}{6} r}$

Ⓒ $\cos^{-1}(-2)$
undefined
↳ $[-1, 1]$

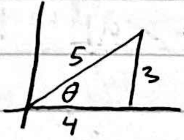
Arctangent = $\tan^{-1} \theta$

Ex 3: Ⓐ $\tan^{-1} \frac{\sqrt{3}}{3}$



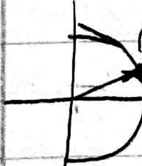
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\frac{1}{2} \cdot \frac{2}{\sqrt{3}} \rightarrow \frac{1}{\sqrt{3}} \rightarrow \boxed{\frac{\sqrt{3}}{3}}$

Ex 7: $\sin(\cos^{-1}\frac{4}{5})$



$\sin(\theta) = \frac{3}{5}$

Ex 6: Ⓐ $\sin(\sin^{-1}\frac{1}{2})$



$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \sin\left(\frac{\pi}{6}\right) \rightarrow \boxed{\frac{1}{2}}$

Ⓑ $\cos^{-1}\left(\cos\frac{5\pi}{2}\right)$

$\frac{5\pi}{2} - \frac{2\pi \cdot 2}{1 \cdot 2} \rightarrow \frac{\pi}{2}$

$\cos^{-1}(0) \rightarrow \boxed{\frac{\pi}{2}}$

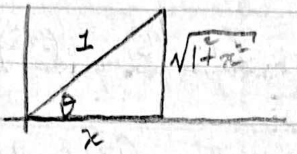
100000001

4.6 part 2

$x^2+b^2=1^2 \rightarrow b^2=1^2-x^2$

Ex 8: cot(arccos x)

cot($\cos^{-1}x$)

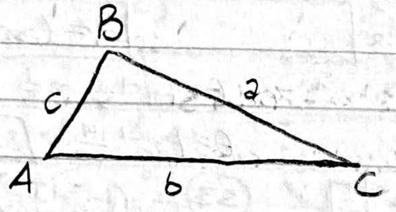


$\cot\theta=\frac{x}{\sqrt{1^2-x^2}} \rightarrow \frac{x\sqrt{1^2-x^2}}{1^2-x^2}$

4.7 Law of Sines & Cosines

Jan. 10, 2023

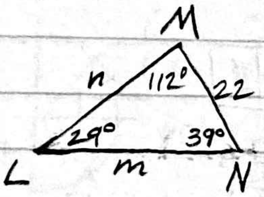
Law of Sines



A, B, and C, then $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

Used in oblique triangles when given AAS, ASA, or SSA

Ex 1:

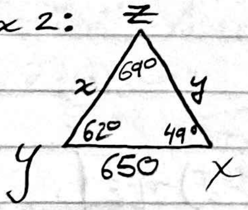


141

$\frac{\sin 29°}{22} \times \frac{\sin 112°}{m}$

$\frac{m\cancel{\sin 29°}}{\cancel{\sin 29°}}=\frac{22\sin 112}{\sin 29°} \rightarrow m=42$

Ex 2:



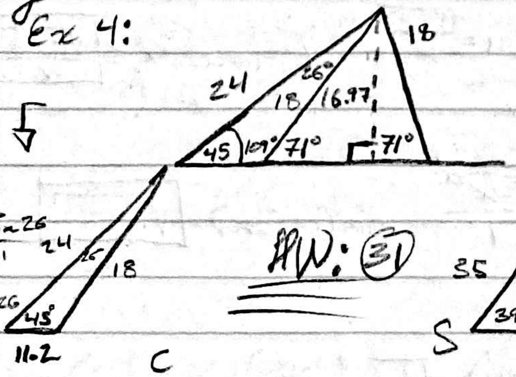
$\frac{\sin 69°}{650}=\frac{\sin 49°}{x} \rightarrow \frac{x\cancel{\sin 69°}}{\cancel{\sin 69°}}=\frac{650\sin 49°}{\sin 69°}$

$x=525$ft

$\frac{\sin 69°}{650} \times \frac{\sin 62°}{y} \rightarrow \frac{y\cancel{\sin 69°}}{\cancel{\sin 69°}}=\frac{650\sin 62°}{\sin 69°}$

$y=$

Ex 4:



$\sin 45=\frac{h}{24} \rightarrow 24\sin 45=h$

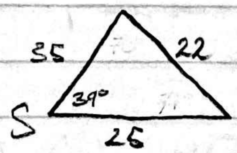
$\frac{\sin 45}{18}=\frac{\sin B}{24}$

$\frac{\sin 45}{18}=\frac{\sin 26}{c'}$

$c'\sin 45=18\sin 26$

$c'=\frac{18\sin 26}{\sin 45}$ $c'=11.2$

HW: (31)



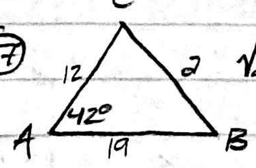
$a^2=b^2+c^2-2bc\cos A$

$22^2=25^2+35^2-2(35(25))\cos S$

$\cos^{-1}\left(\frac{22^2-25^2-35^2}{-2(35(25))}\right)=\cos^{-1}(\cos)S$

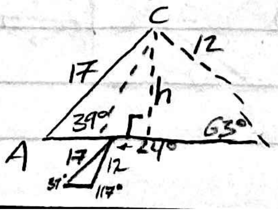
$S=\sim 39°$

(27)



$\sqrt{a^2}=\sqrt{12^2+19^2-2(12)(19)\cos 42°}$

(19) $A=39°$, $a=12$, $b=17$

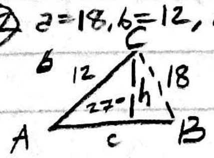


$\sin 39=\frac{h}{17}$

$17\sin 39=h$

$h<12$ ✓ $\rightarrow \approx 10.7$

(12) $a=18$, $b=12$, $A=27°$



SOH

$\sin 27=\frac{h}{12} \rightarrow$

$12\sin 27=h$

$h=5.44$

4.7 part 2

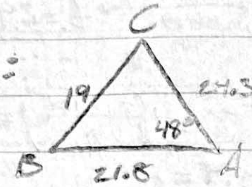
Law of Cosines: Use for oblique triangles for SSS or SAS

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Ex 5:



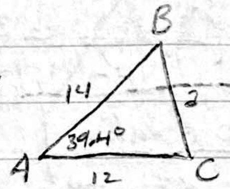
$19^2 = 24.3^2 + 21.8^2 - 2(24.3)(21.8) \cos A$

$19^2 - 24.3^2 - 21.8^2 = -704.73$

$\div (-2(24.3)(21.8)) = 0.665...$

$\cos^{-1}(\text{ANS}) \approx 48° = A$

Ex 6:



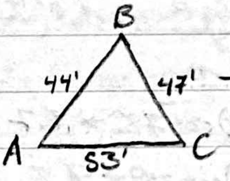
$a^2 = 12^2 + 14^2 - 2(12)(14) \cos 39.4°$

$a = \sqrt{...} \to 8.96 \to 9.0$

Heron's Formula (SSS): If SSS is given, then:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

Ex 7:



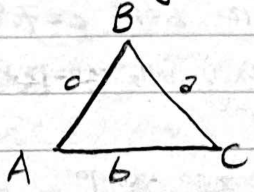
$\to 979'^{2}$

Area of Triangle: If SAS is given, then:

$\text{Area} = \frac{1}{2} bc \sin A$

$\text{Area} = \frac{1}{2} ac \sin B$

$\text{Area} = \frac{1}{2} ab \sin C$



5.1 Trig Identities

Jan. 22. 2023

→ when the left side of an equation equals the right side

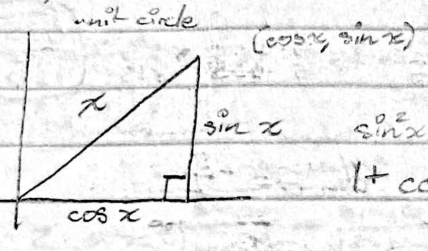
Def: $x \equiv y$ for all values of variable for which both sides are defined

Ex 1: (A) $\cos\theta=\frac{3}{4}$, find $\sec\theta$

$$\sec\theta=\frac{1}{1}\cdot\frac{4}{3}\to\boxed{\frac{4}{3}}$$

(B) $\sec x=\frac{5}{4}$ and $\tan x=\frac{3}{4}$ sin x

$$\frac{\sin x}{4/5}=\frac{3}{4}\to\sin x=\frac{3}{4}\cdot\frac{4}{5}\to\frac{12}{20}\to\frac{6}{10}\to\frac{3}{5}=\sin x$$



Pythagorean Identities:

$\sin^2 x+\cos^2 x=1^2$ $\quad 1+\tan^2 x=\sec^2 x$

$1+\cot^2 x=\csc^2 x$

Ex 2: $\underset{\text{I},\ \boxed{\text{III}}}{\cot\theta=2}$, $\underset{\text{II},\ \boxed{\text{III}}}{\cos\theta<0}$, find $\sin\theta$ & $\cos\theta$

$$\cot^2\theta+1=\csc^2\theta\to 5=\csc^2\theta\to\pm\sqrt{5}=\sqrt{\csc^2}\to\sin\theta=\frac{\pm1}{\sqrt{5}}\to-\frac{\sqrt{5}}{5}$$

~~$1+\tan^2 x=\sec^2 x\to 1+\frac{1}{2}^2=\sec^2\theta\to 1+\frac{1}{4}=\sec^2\theta\to\pm\sqrt{1\frac{1}{4}}=\pm\sec\theta\to\cos\theta=\pm\frac{1}{\sqrt{1\frac{1}{4}}}$~~

~~$\cos\theta=\frac{1}{\sqrt{\frac{5}{4}}}\to\cos\theta=\frac{\sqrt{\frac{5}{4}}}{\frac{5}{4}}$~~ $\quad \cot\theta=\frac{\cos\theta}{-\frac{\sqrt{5}}{5}}\to-\frac{\sqrt{5}}{5}\cot\theta=\cos\theta$

$$-\frac{\sqrt{5}}{5}(2)=\cos\theta\to-\frac{\sqrt{5}}{5}\cdot\frac{2}{1}\to-\frac{2\sqrt{5}}{5}=\cos\theta$$

Ex 3:

$\cos x=-0.75$, find $\sin\left(x-\frac{\pi}{2}\right)$

$$\sin\left(-\frac{\pi}{2}+x\right)$$

$$\sin\left[-1\left(\frac{\pi}{2}-x\right)\right]$$

$$-\sin\left(\frac{\pi}{2}-x\right)=-\cos x=\boxed{0.75}$$

Ex 4: $\frac{1}{\cos x}(1-\sin^2 x)$, simplify:

$$\sec x(1-\sin^2 x)\to\sec x(\cos^2 x)\to\cos x$$

$$\frac{1}{\cos x}=(\cos^2 x)\to\frac{1}{\cos x}\left(\frac{\cos^2 x}{1}\right)\to\frac{\cos^2 x}{\cos x}\to\boxed{\cos x}$$

Ex 5: $\cos x\tan x-\sin x\cos^2 x$

$$\cos x(\tan x-\sin x\cos x)$$

$$\cos x\left(\frac{\sin x}{\cos x}-\frac{\sin x\cos x\cos x}{\cos x}\right)$$

$$\cos x\left(\frac{\sin x-\sin x\cos^2 x}{\cos x}\right)$$

$$\to\sin x-\sin x\cos^2 x$$

$$\sin x(1-\cos^2 x)$$

$$\sin x(\sin^2 x)=\sin^3 x$$

5.1 continued

Ex 6: $\frac{\sec x}{1-\sec x} - \frac{\sec x}{1+\sec x} \rightarrow \frac{1+\sec x(\sec x)}{1+\sec x(1-\sec x)} - \frac{1-\sec x(\sec x)}{1-\sec x(1+\sec x)}$

$$\frac{\cancel{\sec x}+\sec^2 x \cancel{-\sec x}+\sec^2 x}{1-\cancel{\sec x}+\cancel{\sec x}-\sec^2 x} \rightarrow \frac{2\sec^2 x}{1-\sec^2 x} \rightarrow \frac{2\sec^2 x}{-\tan^2 x} = -2\sec^2 x \cot^2 x$$

$$-\frac{2\cos^2}{\cos^2 \sin^2} = -\frac{2}{\sin^2}$$

$\hookrightarrow -2\csc^2 x$

5.1 |||| Pg. 317: 3 5, 9

5.2 Verifying Identities (Trig) Jan. 24. 2023

1. Start w/ complicated side
2. Use Identities (& memorize!)
3. Use algebraic operations

Pythagorean identity ↘

→ ☆ 4. Convert denominator when $1 \pm u$ or $u \pm 1$ to single term using conjugate & ☆
5. If both sides are complicated, work terms to intermediate expression
6. Last resort: convert to $\sin x$ & $\cos x$

Ex 1: $\frac{\tan^2 x+1}{1-\sin^2 x} = \sec^4 x \to \frac{\tan^2 x+1}{\cos^2 x} \to \frac{\sec x}{\cos^2 x} \to \frac{\sec^2 x}{1} \cdot \frac{1}{\cos^2 x} \to \sec^2 x \cdot \sec^2 x$

$\to \boxed{\sec^4 x}$

$(1+x)(1-x)$

$1 \cancel{+x-x} - x^2$

Ex 2: $-2\cot x = \frac{\sin x}{1+\cos x} - \frac{\sin x}{1-\cos x} \to \frac{\cancel{\sin x} - \cos x \sin x}{1-\cos^2 x} - \frac{\cancel{\sin x} + \sin x \cos x}{1-\cos^2 x} \to$

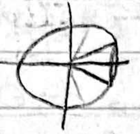
$\frac{-2\cos x \sin x}{\sin^2 x} \to \frac{-2\cos x}{\sin x} \to \boxed{-2\cot x}$

★ ~~Don't~~ divide both sides by trig funcs.
Never

5.3 Solving Trig Equations

Jan. 31. 2023

Ex 1: $5\cos x = 3\cos + \sqrt{3} \to 2\cos x = \sqrt{3} \to \cos x = \frac{\sqrt{3}}{2}$

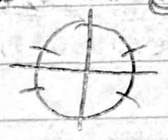


$\to \frac{\pi}{6}, -\frac{\pi}{6} \to \left(\frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}\right), \left(-\frac{\pi}{5} + 2\pi n, n \in \mathbb{Z}\right)$

Ex2: $3\tan^2 x - 4 = -3 \to 3\tan^2 x = 1 \to \tan^2 x = \frac{1}{3} \to \tan x = \pm\sqrt{\frac{1}{3}} \to \boxed{\pm}\frac{1}{\sqrt{3}}$

$\to \tan x = \pm\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \to \pm\frac{\sqrt{3}}{3} \to \frac{1}{2} \cdot \frac{2}{\sqrt{3}} \to \frac{1}{\sqrt{3}} \to \frac{\sqrt{3}}{3}$

$30° \to \frac{\pi}{6}$

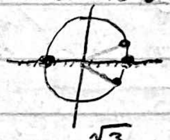


$\frac{\sqrt{3}}{3}$

$\to \frac{\pi}{6} + \pi n, n \in \mathbb{Z} \qquad \to \frac{\pi}{6} + \pi n, n \in \mathbb{Z}$

Ex 3: Ⓐ $2\sin x \cos x = \sqrt{3} \sin x \to 2\sin x \cos x - \sqrt{3}\sin x$

$\to \sin x(2\cos x - \sqrt{3})$



$\boxed{x = 0, \pi, \frac{\pi}{6}, \frac{11\pi}{6}}$

$\sin x = 0 \qquad 2\cos x - \sqrt{3} = 0$

$\cos x - \frac{\sqrt{3}}{2} = 0 \to \cos x = \frac{\sqrt{3}}{2}$

$\to (x+2)(x-1)$

$x^2 \; 1x - 2$

Ⓑ $2\sin^2 x + \sin x - 1 = 0 \to (\sin x + 2)(\sin x - 1)$

$\sin^2 x + \sin x - 2$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1 \qquad \sin x = \frac{1}{2}$

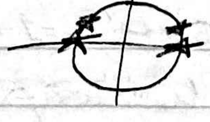
$\boxed{x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$

$60° \to$

$1 - \cos^2 x = \sin^2 x$

Ex5: $\sin^2 x - \sin x + 1 = \cos^2 x \to \sin^2 x - \sin x + 1 - \cos^2 x = 0$

$\to 2\sin^2 x - \sin x \to \sin(2\sin x - 1) = 0$



$\sin x = 0 \qquad \sin x = \frac{1}{2}$

$\boxed{x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}}$

Extraneous!

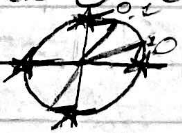
$\to \boxed{x = \frac{\pi}{2}, \pi}$

Ex 6: $\sin x - \cos x = 1 \to (\sin x - \cos x)^2 = (1)^2 \to (\sin x - \cos x)(\sin x - \cos x)$

$\to \sin^2 x - \cos x \sin x - \cos x \sin x + \cos^2 x \to \sin^2 x - 2\sin x \cos x + \cos^2 x$

$1 - 2\cos x \sin x = 1 \to -2\cos x \sin x = 0 \to \cos x \sin x = \frac{0}{-2}$

$\cos x = 0 \qquad \sin x = 0$



$x = \cancel{0}, \frac{\pi}{2}, \pi, \cancel{\frac{3\pi}{2}}$

$\sin x - \cos x = 1 : x = 0$

$\sin 0 - \cos 0 = 1$

$0 - 1 = 1 \to$ ✗

$x = \frac{\pi}{2}$: $\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = 1$

$1 - 0 = 1$

$x = \pi$:

$\sin \pi - \cos \pi = 1$

$0 + 1 = 1$

$x = \frac{3\pi}{2}$: $\sin\frac{3\pi}{2} - \cos\frac{3\pi}{2} = 1$

$-1 - 0 = 1$

$-1 \neq 0$

5.4 Sum & Difference Identities

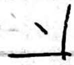
Feb. 5.2023

Exact value of:

Ex 1: ⓐ $\cos 75°$

$\overset{\alpha}{\cos 30°} + \overset{\beta}{\cos 45°} = \cos 30° \cos 45° - \sin 30° \sin 45°$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \to \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \to \frac{\sqrt{6}-\sqrt{2}}{4}$



ⓑ Exact value of: $\tan \frac{11\pi}{12} \Rightarrow \frac{2\pi}{12} + \frac{9\pi}{12} \Rightarrow \left(\overset{\alpha}{\frac{1\pi}{6}} + \overset{\beta}{\frac{3\pi}{4}}\right)\tan$

X $\tan(\alpha+\beta) = \frac{\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{3\pi}{4}\right)}$

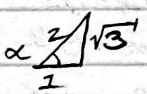
$\Rightarrow \frac{\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}} \to \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}} \to \frac{\frac{\sqrt{2}+\sqrt{6}}{2}}{\frac{\sqrt{6}-\sqrt{2}}{2}} \to \frac{\sqrt{2}+\sqrt{6}}{2} \cdot \frac{2}{\sqrt{6}-\sqrt{2}} \to$

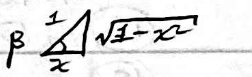
$\to \boxed{\frac{\sqrt{2}-\sqrt{6}}{\sqrt{6}+\sqrt{2}}}$

$b = \sqrt{1+x^2}$

$x^2+b^2=1^2$

Ex 3: Ⓑ $\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \to \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \Rightarrow \boxed{\frac{\sqrt{6}+\sqrt{2}}{2}}$





Ex 4: $\cos\left(\arcsin\frac{\sqrt{3}}{2} + \arccos x\right) \to \cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1} x\right)$

$\frac{1}{2} \cdot \frac{x}{1} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{1-x^2}}{1} \to \frac{x}{2} - \frac{\sqrt{3-3x^2}}{2} \to \boxed{\frac{x-\sqrt{3-3x^2}}{2}}$

verify:

Ex 5: $\cos(-\theta) = \cos\theta \to \cos(\overset{\alpha}{0} - \overset{\beta}{\theta}) = \overset{\alpha}{\cos}\,\overset{\beta}{\cos} + \overset{\alpha}{\sin}\,\overset{\beta}{\sin}$

$\cos 0 \cos\theta + \sin 0 \sin\theta \to 1\cos\theta + \cancel{0 \sin\theta}$

$\Rightarrow \boxed{\cos\theta}$

Ex 7: $\sin\left(\overset{\alpha}{x} + \overset{\beta}{\frac{\pi}{4}}\right) - \sin\left(\overset{\alpha}{x} - \overset{\beta}{\frac{\pi}{4}}\right) = 0$

$\left.\begin{array}{l} \sin x \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos x \\ -\sin x \cos -\frac{\pi}{4} + \sin\frac{\pi}{4}\cos x \end{array}\right\}$ $2\sin\frac{\pi}{4}\cos x = 0$

$\frac{2}{1} \cdot \frac{\sqrt{2}}{2} \cdot \cos x = 0 \to \sqrt{2} \cdot \cos x$

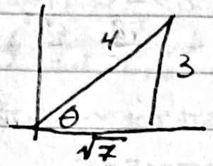
5.5 Multiple-Angle & Product to sum Identities

Feb. 7. '24

Ex1: $\sin\theta = \frac{3}{4}, \left(0, \frac{\pi}{2}\right)$ $\underline{|\,I}$ → All positive values for triangle

$a+b^2=16$

$b^2=7$



$\sin 2\theta = 2\sin\cos$

$2 \cdot \frac{3}{4} \cdot \frac{\sqrt{7}}{4} \to \frac{6\sqrt{7}}{16} \to \boxed{\frac{3\sqrt{7}}{8}} = \sin 2\theta$

$\cos 2\theta = \frac{7}{16} - \frac{9}{16} \to -\frac{2}{16} \to \boxed{-\frac{1}{8}} = \cos 2\theta$

$\tan 2\theta = \frac{2 \cdot \frac{3}{4} \cdot \frac{\sqrt{7}}{4}}{\frac{7}{16} - \frac{9}{16}} \to \frac{\frac{3\sqrt{7}}{8}}{-\frac{1}{8}} \to \frac{3\sqrt{7}}{8} \cdot -\frac{8}{1} \to \boxed{-3\sqrt{7}} = \tan 2\theta$

Ex2: $\cos 2\theta - \cos\theta = 2, [0, 2\pi)$

$\cos^2 - \sin^2$ $\to 2\cos^2\theta - 1 - \cos\theta = 2 \to 2\cos^2 - \cos - 3 = 0$

* $2\cos^2 - 1$ $\to \cos^2 - \cos - 6 \to (\cos - 3)(\cos + 2) \to (2\cos - 3)(\cos + 1)$

$1 - 2\sin^2$ $\cos\theta = \cancel{\frac{3}{2}}, -1 \to \boxed{\theta = \pi}$

★ Ex3: $\sin^4\theta = (\sin^2\theta)^2 \to \left(\frac{1-\cos 2\theta}{2}\right)^2 \to \left(\frac{1-\cos 2\theta}{2}\right)\left(\frac{1-\cos 2\theta}{2}\right)$

$\to \frac{1 - 2\cos 2\theta + \cos^2 2\theta}{4} \to \frac{1 - 2\cos 2\theta + \left(\frac{1+\cos 4\theta}{2}\right)}{4}$

$\to \frac{\frac{2}{2} - \frac{4\cos 2\theta}{2} + \frac{1+\cos 4\theta}{2}}{4} \to \frac{2 - 4\cos 2\theta + 1 + \cos 4\theta}{} \to \frac{\frac{3 - 4\cos 2\theta + \cos 4\theta}{2}}{4}$

$\to \frac{3 - 4\cos 2\theta + \cos 4\theta}{2} \cdot \frac{1}{4} \to \boxed{\frac{3 - 4\cos 2\theta + \cos 4\theta}{8}}$

Ex4: $\sin^2\theta + \cos 2\theta - \cos\theta = 0 \to \frac{1-\cos 2\theta}{2} + \cos 2\theta - \cos\theta = 0$

$\to \frac{1-\cos 2\theta}{2} + \frac{2\cos 2\theta}{2} - \frac{2\cos\theta}{2} = 0 \to \frac{1 + \cos 2\theta - 2\cos\theta}{2} = 0$

$\to 1 + \cos 2\theta - 2\cos\theta = 0 \to 1 + 2\cos^2\theta - 1 - 2\cos\theta = 0 \to 2\cos^2\theta - 2\cos\theta = 0$

$\to 2\cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0, \cos\theta = 1$ $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \to + 2\pi n, n \in \mathbb{Z}$

No. 7.1

Date Feb 21 2024

7.1 Parabolas

Conic Sections: when a plane intersects a double-napped right cone

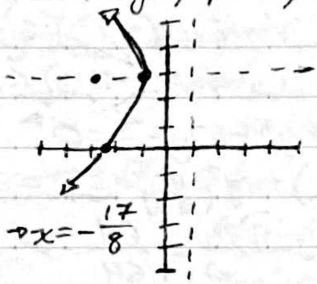
Locus: set of all points that fulfill a geometric property

Parabola: Locus that is equidistant from a fixed point called the focus & line called the directrix

Ex 1: $(y-3)^2 = -8(x+1)$ $p=-2$ opens left/right, $p<0$, left

$v: (-1, 3)$ axis: $y=3$

$f: (-3, 3)$ direct: $x=1$



3rd point → $y=0$ → $(0-3)^2 = -8(x+1)$

$9 = -8x - 8 \rightarrow 17 = -8x \rightarrow x = -\frac{17}{8}$

Ex 3: Completing the Square: $\overset{\downarrow}{x^2} - 8x - y = -18$ $\left(\frac{b}{2}\right)^2 = \underline{\quad}$

↕ $(x-h)^2 = 4p(y-k)$ $x^2 (-8)x + \underline{16} = y - 18 + \underline{16} \rightarrow (x-4)^2 = y-2$

$\rightarrow (x-4)^2 = 1(y-2)$

Ex 4: ② $f(2,1)$ $v\underline{(-5,1)}$ ↔ right $p=7$

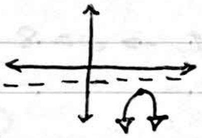
$(y-1)^2 = 28(x+5)$

distance between 2 points:

$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Δx Δy

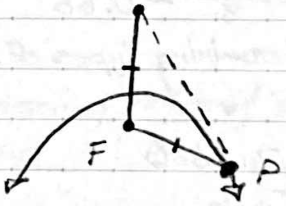
② $v(3,-2)$ $d: y=-1$ ↕ down $p=-1$



$(x-3)^2 = -4(y+2)$

Line Tangent to a Parabola:

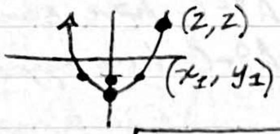
- Line "l" is tangent at "P" and forms isosceles triangle
- "P" to focus is 1 leg
- "l" to focus is other



Ex 5: tangent to $y = x^2 - 2$ at $(2,2)$

$y + 2 = x^2 \rightarrow p = \frac{1}{4}$

$(x+0)^2 = 1(y+2) \rightarrow v(0,-2)$



$f(\underset{x_2}{0}, \underset{y_2}{-1.75})$ $D = \sqrt{(0-2)^2 + (-1.75-2)^2} = 4.25$

(2,2) (0,−6)

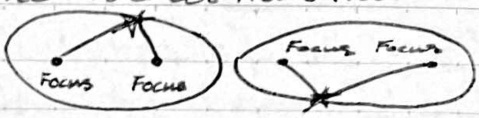
No.

Date Feb 23 2024

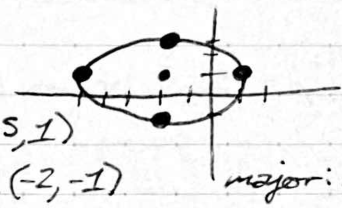
7.2 Ellipses & Circles

Ellipse: A locus of points such that the sum of the distances from two fixed points, called foci, is constant.

SAME!



Ex1: ⓐ $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$



Center: $(-2,1)$ ¦ Vertices: $(1,1), (-5,1)$

$a: 3$ $b: 2$ ¦ Co-vert: $(-2,3), (-2,-1)$

foci: $9 = 4 + c^2 \rightarrow 5 = c^2$

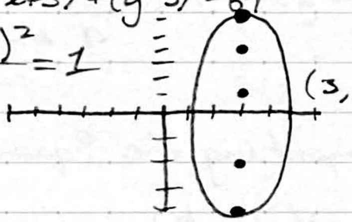
$c = \sqrt{5} \rightarrow (-2 \pm \sqrt{5}, 1)$

major: $y = 1$ minor: $x = -2$

ⓑ $4x^2 + 24x + y^2 - 10y - 3 = 0$

$4(x^2 \left(\frac{+6}{2}\right)^2 x + \underline{9}) + y^2 \left(\frac{-10}{2}\right)^2 y + \underline{25} = 3 + \underline{36} + \underline{25} \rightarrow 4(x+3)^2 + (y-5)^2 = 64$

$\rightarrow \frac{4(x+3)^2 + 1(y-5)^2}{64} = \frac{64}{64} \rightarrow \frac{(y-5)^2}{64} + \frac{(x+3)^2}{16} = 1$



$(3,1)$ = cen

Ex 2: ⓑ vert: $(3,-4), (3,6)$ foci: $(3,4), (3,-2)$

$c = 3$ $a = 5$ $b = 4$ $25 = 9 + b^2 \rightarrow 16 = b^2 \rightarrow b = 4$

$\frac{(y-1)^2}{25} + \frac{(x-3)^2}{16} = 1$

Eccentricity: Ratio of $c \rightarrow a$. Always between 0 & 1 for a ellipse & determines how "circular" or "streched" the ellipse will be.

$$e = \frac{c}{a}$$

Ex 3: $\frac{(x-4)^2}{64} + \frac{(y-3)^2}{36} = 1 \rightarrow a = 8$ $b = 6$ $64 = 36 + c^2 \rightarrow 28 = c^2 \rightarrow \sqrt{28} = c$

$e = \frac{\sqrt{28}}{8} \approx 0.66$

Determining Types of Conics

$\underset{\sim}{Ax^2} + Bxy + \underset{\sim}{Cy^2} + Dx + Ey + F = 0$

parabola	A=0 or C=0, <u>not</u> both
Circle	A=C
Ellipse	A & C have same sign, A≠C
Hyperbola	A & C have opposite signs

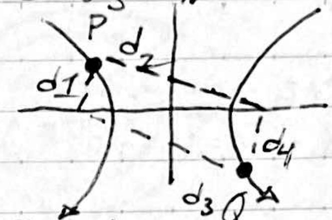
Table only useful when there is no "B"

No.

Date Feb 26 2024

7.3 Hyperbolas

Hyperbola: Locus of points such that the abs$(d_1 - d_2)$ = abs$(d_3 - d_4)$



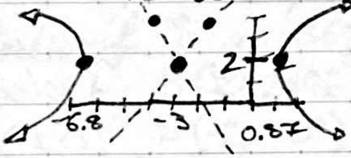
Ex 2: $4x^2 - y^2 + 24x + 4y = 28 \rightarrow 4(x^2 + (\frac{6}{2})^2 x + \underline{9}) - 1(y^2 (-\frac{4}{2})^2 y + \underline{4}) = 28 + \underline{36} - \underline{4}$

$\rightarrow 4(x+3)^2 - (y-2)^2 = 60 \rightarrow \frac{(x+3)^2}{15} - \frac{(y-2)^2}{60} = 1$ center$(-3, 2)$

$a^2 \rightarrow 15$

vert.: $(-3 \pm \sqrt{15}, 2)$

$a = \sqrt{15}$ $b = \sqrt{60} = 2\sqrt{15}$ $15 + 60 = c^2 \rightarrow \sqrt{75} = c \rightarrow 5\sqrt{3} = c$ foci.: $(-3 \pm 5\sqrt{3}, 2)$



asy.: $y - 2 = \pm\frac{2\sqrt{15}}{\sqrt{15}}(x+3)$

↳ or: $y - 2 = \pm 2(x+3)$

Ex 3: ⑥ vert.: $(-3, 10), (-3, -2)$ conj. ax.: 6 $\rightarrow b = 3$ center: $(-3, 4)$ $a = 6$

$36 + 9 = c^2 \rightarrow 45 = c^2 \rightarrow c = \sqrt{45}$ $\frac{(y-4)^2}{36} - \frac{(x+3)^2}{9} = 1$

Eccentricity of hyperbolas: $e > 1$ $e = \frac{c}{a}$ $a^2 + b^2 = c^2$

7.5 Parametric Equations

Feb.28.2024

Def: Graph representing 2 continuous functions } "Parametric Curve"

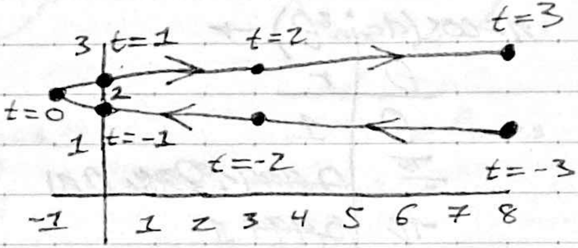
Parametric equations: $x = f(t)$; $y = g(t)$

Parameter: Values representing time or angle measures.

Orientation/"Direction": Plotting points in the order of increasing values of "t" for a parametric curve. Orientation represented by arrows

*Ex 1: $x = t^2 - 1$, $y = \frac{t}{4} + 2$, interval $-3 \le t \le 3$

t	x	y
-3	8	1.25
-2	3	1.5
-1	0	1.75
0	-1	2
1	0	2.25
2	3	2.5
3	8	2.75



No.

Date

7.5 continued

Ex 2: $y=2t$, $x=t^2+2$ in rectangular form

$x=t^2+2 \rightarrow x-2=t^2 \rightarrow \pm\sqrt{x-2}=t$ ☆

☆ $y=2t \rightarrow y=2(\pm\sqrt{x-2}) \rightarrow y=\pm 2\sqrt{x-2}$

Ex 3: Restrictions $\left[y=\frac{1}{2t}\right]$ & $\boxed{\left[x=\sqrt{t+1}\right]}$ domain restrictions come from this component's range values

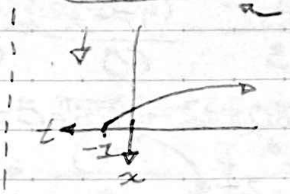
$x=\sqrt{t+1}$

$x^2=t+1$

$x^2-1=t$ ☆

$y=\frac{1}{2(x^2-1)}$

$y=\frac{1}{2x^2-2}$



$x \geq 0$

$2x^2-2 \neq 0,\ 2x^2 \neq 2,\ x^2 \neq 1,\ x \neq \pm 1$

Choose most restricted domain

$\{x \mid x \geq 0, x \neq 1, x \in \mathbb{R}\}$

$[0,1) \cup (1,\infty)$

Ex 4: ☆ $y=5\sin\theta$ & $x=3\cos\theta$

$y=5\sin\theta \rightarrow \frac{y}{5}=\sin\theta$

$x=3\cos\theta \rightarrow \frac{x}{3}=\cos\theta$

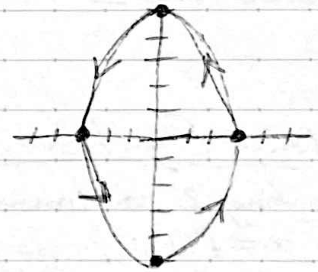
$\sin^2+\cos^2=1$

$\left(\frac{y}{5}\right)^2+\left(\frac{x}{3}\right)^2=1$

$\frac{y^2}{25}+\frac{x^2}{9}=1$

θ	x	y
0	3	0
π/2	0	5
π	-3	0
3π/2	0	-5
2π	3	0

$\left.\right\} \begin{array}{l} x=3\cos\theta \\ y=5\sin\theta \end{array}$



Polar project table:

$((\sqrt{\cos^2 2t})\cos t),((\sqrt{0.6\cos 2t})\sin t-1.5)$ = parametric equation

t	x	y
0	1	~~0~~ -1.5
π/2	0 ~~0.999~~	~~0.021~~, -1.479
π	-1 ~~0.996~~	~~0.042~~ -1.458
3π/2	0 ~~0.991~~	~~0.063~~ -1.437
2π	~~0.985~~	~~0.084~~ -1.416

0 ~~0.77~~

$\sqrt{\cos^2 2t}\ \cos t = x$

$\sqrt{0.6\cos 2t}\ \sin t - 1.5 = y$

$\sqrt{\cos^2 2\emptyset}\ \cos\emptyset = 1$

x
1
0
-1
0
1

$r=\cos(d\sin^2\theta) \rightarrow$

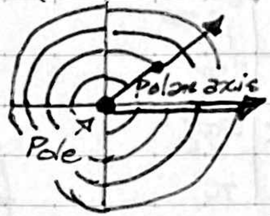
θ	r
0	1
π/2	~~0.999~~ ~~[illegible]~~ 0.91
π	~~0.999~~ 1
3π/2	~~0.999~~ 0.91
2π	~~0.999~~ 1

No.

9.1 Polar Coordinates

Date Mon 1 2024

Polar Coordinate System: Coordinate system using "r" (distance from center) & θ (angle from polar axis)



Ex: $(2, 30°)$

Polar coordinates: (r, θ)

$+r$: point lies on terminal side of θ

$-r$: point lies on the opposite ray of the terminal side of θ

Multiple point representation: $(r, \theta) = (r, \theta \pm 360°)$ & $(r, \theta \pm 2\pi)$

$(r, \theta) = (-r, \theta \pm 180°)$ & $(-r, \theta \pm \pi)$

Ex 3: 4 different points if θ is $-360° < \theta < 360°$

S $(2, 210°) \rightarrow (2, -150°)$

S $(-2, 30°) \rightarrow (-2, -330)$

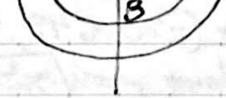
Ex 4: Ⓐ $r = 2.5$ Ⓑ $\theta = \frac{5\pi}{6}$

Polar distance formula: $d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

↳ Pay attention to degree/radian mode

Ex 5: $A(\overset{r_1}{8}, \overset{\theta_2}{60°})$ & $B(\overset{r_2}{4}, \overset{\theta_1}{300°})$

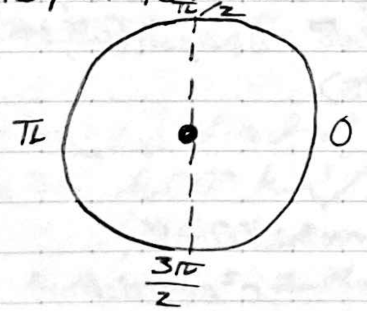
$d = \sqrt{8^2 + 4^2 - 2(8)(4)\cos(300-60)} = 10.583$



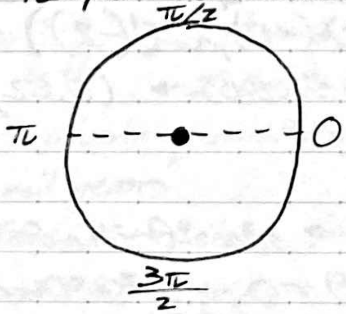
9.2 Graphs of Polar Equations

Symmetries:

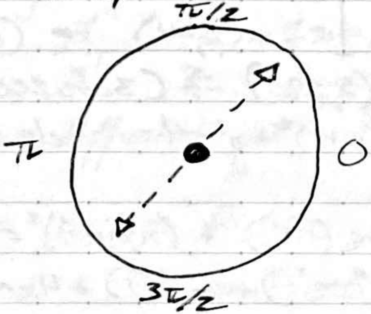
Respect to Line $\theta=\frac{\pi}{2}$



Respect to Polar Axis



Respect to Pole



r= sine funcs

r=cosine funcs

$(-r,\theta)$ or $(r,\pi+\theta)$

Ex 1: $r=3\cos\theta$

diameter=3

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
r	3	2.6	2.1	1.5	0	-1.5	-2.1	-2.6	-3

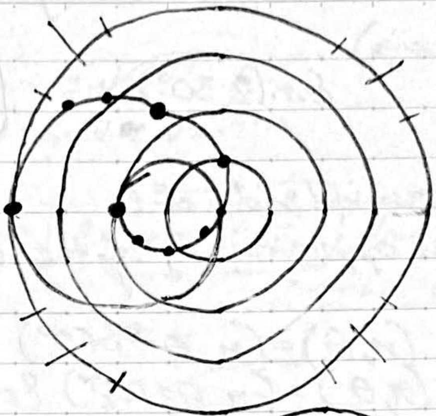
No.

Date

→ Limogon w/ loop

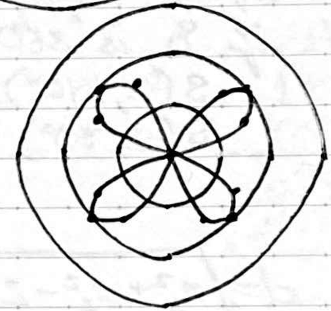
Ex 2: $r = 1 - 3\cos\theta$

r	θ	r	θ
-2	0	3.1	$3\pi/4$
-1.6	$\pi/6$	3.6	$5\pi/6$
-1.1	$\pi/4$	4	π
-0.5	$\pi/3$		
1	$\pi/2$		
2.5	$\frac{2\pi}{3}$		



Ex: $r = 2\sin 2\theta$ → 4 petals

r	θ	r	θ
0	$-\pi/2$	1.7	$\pi/6$
-1.7	$-\pi/3$	2	$\pi/4$
-2	$-\pi/4$	1.7	$\pi/3$
-1.7	$-\pi/6$	0	$\pi/2$
0	0		



9.3 Polar & Rectangular Forms of Equations

★Pay attention to quadrants → are there negatives?

convert: $x = r\cos\theta$ $\quad (r,\theta) \to (r\cos\theta, r\sin\theta) \to (x,y)$

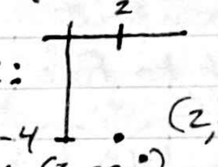
$y = r\sin\theta$

Ex1: ③ $D\left(2, \frac{\pi}{3}\right) \to \left(2\cos\frac{\pi}{3}, 2\sin\frac{\pi}{3}\right) \to (1, \sqrt{3})$

⑥ $F(-5, 45^\circ) \to (-5\cos 45, -5\sin 45) \to \left(-\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$

Rectangular to polar: $r = \sqrt{x^2+y^2}$ $\quad (x,y) \to \left(\sqrt{x^2+y^2}, \tan^{-1}\frac{y}{x}\right) \to (r,\theta)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Ex 2: 

1st point $\quad$ same point

$(2,-4) \to \left(\sqrt{2^2+(-4^2)}, \tan^{-1}(-2)\right) \to (2\sqrt{5}, -63.4^\circ)$ or $(2\sqrt{5}, 296.6^\circ)$

Ex 3: $(3, 280^\circ) \to (3\cos 280, 3\sin 280) \to (0.52, -2.95)$

Ex 4: ⑦ $(x+2)^2 + y^2 = 4$ circle

$(r\cos\theta+2)^2 + (r\sin\theta)^2 = 4 \to r^2\cos^2\theta + 4r\cos\theta + \cancel{4} + r^2\sin^2\theta = \cancel{4}$

$\to r^2(\cos^2\theta + \sin^2\theta) + 4r\cos\theta = 0 \to r^2 + 4r\cos\theta = 0 \to r^2 = -4r\cos\theta$

$r = -4\cos\theta$

⑥ $2xy = 4 \to \quad \to 2(r\cos\theta)(r\sin\theta) = 4 \to 2\sin\theta\cos\theta \cdot r^2 = 4$

$\to \sin 2\theta \cdot r^2 = 4 \to r^2 = \frac{4}{\sin 2\theta}$

No.

Date Mar 11 2024

Ex 5: ⓐ $\theta=\frac{\pi}{4} \rightarrow \tan^{-1}\frac{y}{x}=\frac{\pi}{4} \rightarrow \frac{y}{x}=\tan\left(\frac{\pi}{4}\right)$

$\frac{y}{x}=1 \rightarrow y=x$

$y=r\sin\theta$

$\Leftrightarrow \frac{y}{r}=\sin\theta$

ⓑ $r=5 \rightarrow \sqrt{x^2+y^2}=5 \rightarrow x^2+y^2=25$

ⓒ $r=2\sin\theta \rightarrow r=2\frac{y}{r} \rightarrow r^2=2y$

$\rightarrow x^2+y^2=2y \rightarrow x^2+\left(y^2-2y+\underline{1}\right)=0+\underline{1}$ $\left(\frac{-2}{2}\right)^2$

$\rightarrow x^2+(y-1)^2=1$

9.4 Polar Forms of Conic Sections

Eccentricities

Ellipse: $0 < e < 1$ Parabola: $e = 1$ Hyperbola: $e > 1$

Polar Equations of Conics: $\boxed{r = \frac{ed}{1 \pm \substack{e\sin\theta \\ e\cos\theta}}}$

↳All depends on directrix: if $x = \pm d$ → cos, if $y = \pm d$ → sin

Ex: $r = \frac{10}{3 + 2\cos\theta} \rightarrow r = \frac{\frac{10}{3}}{1 + \frac{2}{3}\cos\theta} \rightarrow e: \frac{2}{3}$; ellipse

$\rightarrow \frac{2}{3} d = \frac{10}{3} \rightarrow d = 5 \quad x = 5$

Statistics Overview & Prep

Stat: Ratio, Proportions, Units, Percentage

Ratios: Comparison of 2 quantities. Expressed as $a:b$ or $\frac{a}{b}$

Ex 1: 240 people → 5 adults to 1 children

(A) 40 children

Proportions: Equality of 2 ratios. Helps find unknown quantities.

Ex 2:

$1'' : 3'$ 3.5×5 $\frac{a}{b} = \frac{x}{y} \rightarrow bx = ay$ Cross multiply!

(D) 157.5 sqft $\frac{1''}{3ft} = \frac{3.5''}{x\,ft} \rightarrow 1x = 10.5$ $\frac{1''}{3'} = \frac{5''}{x'} \rightarrow 1x = 15$

10.5×15

Rates: Quotient of ratio where quantities have different units

Ex 3: 1,060 km / 10.3 mil years

1,060 km → 1,060/1.6 → miles = 662.5 or 663   (C) 663

$\frac{1{,}060\,km}{1} \cdot \frac{1\,miles}{1.6\,km} \rightarrow \frac{1{,}060}{1.6} \approx 663$ miles

Ex 4: a: $30\,mi^2 \rightarrow 370$ people/mi^2 b: $50\,mi^2 \rightarrow 290$ people/mi^2

$\frac{370\,people}{mi^2}$ $80\,mi^2$ 320 people/mi^2

660 people/mi^2

No.

Date Apr 3 2024

Stat: Representations of Data

Scatterplots: Two variables as points in xy plane.

→ Correlation: Relationship between 2 variables → Use "r" as variable


Strong Positive


Weak Positive


Strong Negative


Weak Negative

Ex: Ⓐ 89 Ⓑ 8 Ⓒ $\frac{-32}{1} \rightarrow \frac{\text{pints}}{\$1}$ → decrease of 32 pints sold per increase $1

Ⓓ $y=233-32(1.5) \rightarrow 185$

$\overset{t}{}$ $\overset{d}{}$

Ex 2: $(16.3, 615) \rightarrow y = t + 33t \rightarrow$ Ⓓ

$(25, 910)$ $y=300+33t$

Ex 3: Ⓓ Ex 4: Ⓐ Ex: Ⓑ

Stat: Statistics

Mean: Average $\bar{x}$=mean Med=median Outliers: Data point that is very different from the rest

Median: Exact middle of a set of data

Range: Largest data point − smallest = range size

Ex: 1: A: Mean: 12 Median: 32.5 Range: 100

B: 10

Ex: 2: Ⓒ Median: 26; Mean: 32

Standard deviation: Value representing distance of data from average

→ Always ≥ 0 → Bigger number = less concentrated data

Stat: 2-Way Tables & Probability

2-Way Table: Summary of data broken into tables

Probability: Calculated as fraction

$$\text{Probability} = \frac{\#\text{ of favorable outcomes}}{\#\text{ of total outcomes}}$$

Polar Graph Transformations

$r=2+3\cos\theta \rightarrow \underline{x=2,\ y=-3 \text{ is what we want for the center}}$

$x=r\cos\theta$ $y=r\sin\theta$

$x=(2+3\cos\theta)\cos\theta$ $y=(2+3\cos\theta)\sin\theta$

$x=(2+3\cos\theta)\cos\theta \underline{+2}$ $y=(2+3\cos\theta)\sin\theta \underline{-3}$

$((2+3\cos t)\cos t+2, (2+3\cos t)\sin t-3))$

Ex: $r=5\cos 2\theta \to 2 \downarrow 3$ $x=r\cos\theta$ $y=r\sin\theta$

$x=(5\cos 2\theta)\cos\theta$ $y=(5\cos 2\theta)\sin\theta$

Use t instead of θ in Desmos for parametric equations

8.1 Vectors

No rulers nor protractor!

Apr. 19. 2024

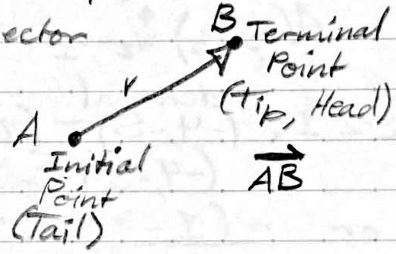
Def: Quantity with both magnitude & direction

Direction: Angle between positive x-axis and vector

Magnitude: Length of vector, |v|

Scalar: Real number, k, represents many physical quantities, such as speed

Standard position: Initial point at origin

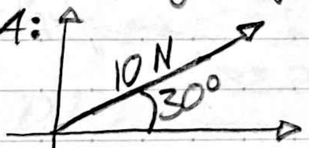


$\overrightarrow{AB}$

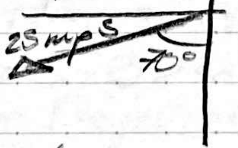
Quadrant Bearing: Between 0° & 90° measured from North-South line.

True Bearing: Angle measured from clockwise North (given in 3 digits)

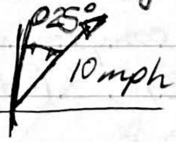
Ex 2: A:



B:



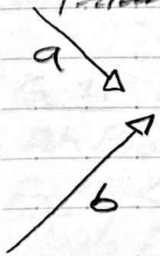
C:

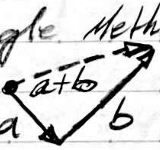


Opposite Vectors: Same magnitude, opposite direction:

Resultant: Single vector composed of sum of 2 or more vectors

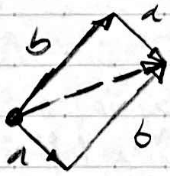
Triangle Method (Tip to tail) (initial to terminal)



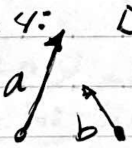


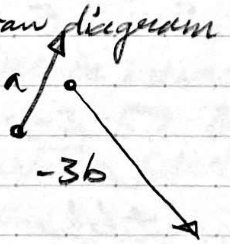
New vector between initial 1 to terminal 2

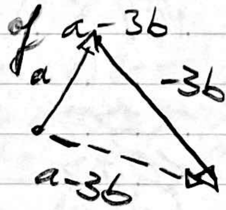
Parallelogram Method (Tail to tail)



Ex 4: Draw diagram of a-3b







if $k>0 \to$ same direction as v

if $k<0 \to$ opposite direction of v

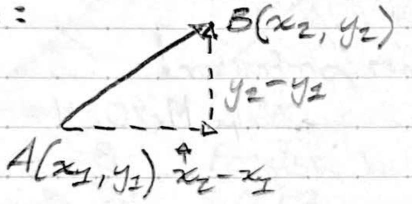
No.

Date Apr 22 '24 8.2 Vectors on the Coordinate Plane

__Component Form__: $\langle x, y \rangle$ where x & y are rectangular components of the vector

Component Form: $\langle x_2 - x_1, y_2 - y_1 \rangle$ ★ Terminal → Initial
→ Terminal − initial

Ex:



Ex 1: $\overset{\text{terminal}}{(-4,-5)} - \overset{\text{initial}}{(1,-3)} = \langle -5,-2 \rangle$ component form

$(-4,-5)$ terminal

or $\underline{-(1,-3)}$ initial

$\langle -5,-2 \rangle$ component

__Magnitude__: $\|v\|$ or $|v| = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$ $|\overrightarrow{AB}|$ or $\|\overrightarrow{AB}\|$
or given component form: $\sqrt{x^2+y^2}$ → magnitude

Ex 2: $(1,3)i$, $(-4,-5)t$ → $\langle -5,-2 \rangle$ → $|v| = 5.38$

Ex 3: B: $3y - 2\cancel{z}$ in $w = \langle 2,-5 \rangle$ $y = \langle 2,0 \rangle$ $z = \langle -1,-4 \rangle$

$\langle 6,0 \rangle$

$\underline{-\langle -2,-8 \rangle}$

$\langle 8,8 \rangle$

__Unit Vector__: Vector with magnitude of 1 unit

To find: divide vector by its magnitude → $u = \frac{v}{|v|}$

Ex 4: $v = \langle 4,-2 \rangle$ $\sqrt{16+4} = |v|$ → $\sqrt{20}$ → $2\sqrt{5}$

$$\left\langle \frac{4}{2\sqrt{5}}, -\frac{2}{2\sqrt{5}} \right\rangle \rightarrow \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle \rightarrow \left\langle \frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5} \right\rangle$$

Standard Unit Vectors:

Vector in positive x direction: $i = \langle 1,0 \rangle$

Vector in positive y direction $j = \langle 0,1 \rangle$

★ Any vector $\langle a,b \rangle$ can be written as $ai + bj$

Linear Combination: Vector sum of $ai + bj$

Ex: $\langle 2,3 \rangle = 2i + 3j = 2\langle 1,0 \rangle + 3\langle 0,1 \rangle$

$= \langle 2,0 \rangle + \langle 0,3 \rangle \rightarrow \langle 2,3 \rangle$

Ex 5: $\langle -3,-3 \rangle$ ini., $\langle 2,6 \rangle$ term. → $5i + 9j$

2, 6

$\underline{-\,-3,-3}$

5, 9

No.

Date Apr 22 '24

Component Form: If given magnitude & direction:

$v = \langle |v|\cos\theta, |v|\sin\theta \rangle$

Ex 5: $v = |v| = 7$, $\theta = 60°$ find component form

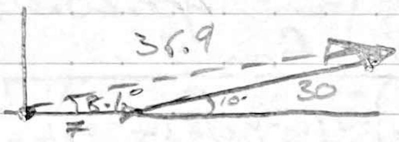
$v = \langle 7\cos 60°, 7\sin 60° \rangle$

$v = \langle 7 \cdot \frac{1}{2}, 7 \cdot \frac{\sqrt{3}}{2} \rangle$ or $\langle \frac{7}{2}, \frac{7\sqrt{3}}{2} \rangle$

Direction Angle: $\tan^{-1}\frac{y}{x}$ * Double check quadrant *

Ex 7: $\tan^{-1}\left(\frac{9}{2}\right) \rightarrow 77.5°$

Ex 8:



player: $\langle 7\cos 0°, 7\sin 0° \rangle$

ball: $+\underline{\langle 30\cos 10°, 30\sin 10° \rangle}$

resultant $\langle 36.544, 5.209 \rangle$

magn $|v| = \sqrt{36.544^2 + 5.209^2} = 36.9$ m/s

direction $= \tan^{-1}\frac{5.209}{36.544} = 8.1123°$

8.3 Dot Products & Vector Projections

Apr. 24. 2024

Dot Product: $a = \langle a_1, a_2 \rangle$ & $b = \langle b_1, b_2 \rangle$

$a \cdot b = a_1 b_1 + a_2 b_2$ → gives 1 constant

Orthogonal Vectors: 2 vectors with dot product = 0

→ Vectors are perpendicular

Ex 1: $u = \langle -3, 4 \rangle$ $v = \langle 3, 6 \rangle \rightarrow -9 + 24 = 15$ does not equal 0

Dot Product & Vector Magnitude Relationship: $u \cdot u = |u|^2$

Ex 2: $\langle -6, 5 \rangle \rightarrow 36 + 25 \rightarrow 61 \rightarrow |a| = \sqrt{61} \approx 7.81$

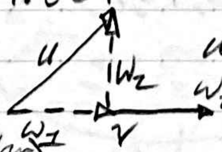
Angle Between 2 Vectors: $\theta = \cos^{-1}\dfrac{a \cdot b}{|a||b|}$

→ θ must be between 0 & π

Ex 3: $u = \langle -3, -5 \rangle$ & $v = \langle 2, -3 \rangle$ $u \cdot v = -6 + 15 = 9$

$|u| = \sqrt{34}$ $|v| = \sqrt{13}$ $\theta = 64.654°$

Vector Projection: $\underset{w_1}{\text{proj}_v u} = \underbrace{\left(\frac{u \cdot v}{|v|^2}\right)}_{\text{becomes } k \text{ (scalar)}} v$



$u = w_1 + w_2$

$w_2 = u - w_1$

- u & v are vectors
- w_1 & w_2 are rect. components of u
- w_1 is the vector projection parallel to v
- w_2 is the component perpendicular to v

Ex 4: $u = \langle -1, 5 \rangle$ & $v = \langle 4, 6 \rangle \rightarrow -4 + 30 = 26$ $|v|^2 = 52$

$\left(\frac{26}{52}\right)\langle 4, 6 \rangle \rightarrow \left(\frac{1}{2}\right)\langle 4, 6 \rangle \rightarrow \boxed{\langle 2, 3 \rangle} = w_2$

$u = \langle -1, 5 \rangle \rightarrow w_1 = \langle -1, 5 \rangle - \langle 2, 3 \rangle \rightarrow w_1 = \boxed{\langle -3, 2 \rangle}$

No.

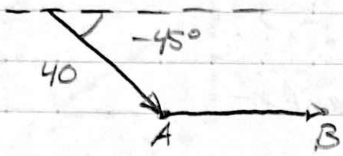
Date Apr 24 2024 **Work Formula:** Constant force moves object from A to B

$W = F \cdot \overrightarrow{AB}$

$\overrightarrow{AB}$: directed force

F: constant force

Ex 7:



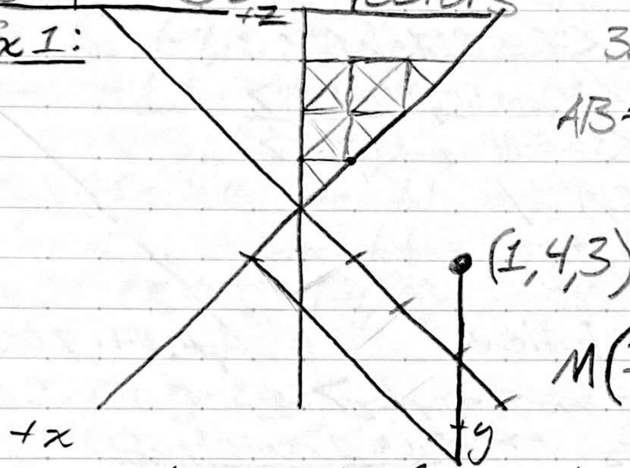
$\langle 40\cos -45, 40\sin -45 \rangle \circ \langle 12, 0 \rangle$

339.411 joules

8.4 3D Vectors

Apr. 26. 2023

Ex 1:



3D distance formula:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

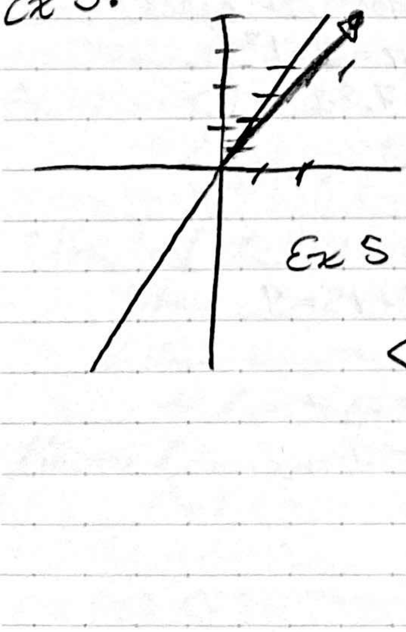
3D Midpoint formula:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Ex 2: $(30, 40, 10), (70, 80, 20) \rightarrow \sqrt{(70-30)^2 + (80-40)^2 + (20-10)^2} = 57.445'$

$\rightarrow \frac{70+30}{2}, \frac{80+40}{2}, \frac{20+10}{2} \rightarrow M(50, 60, 15)$

Ex 3:



Ex 4: $A(3, -2, -1)$ $B(1, 5, -3)$

component: $\langle -2, 7, -2 \rangle$

$|\overrightarrow{AB}| = \sqrt{4 + 49 + 4} \rightarrow \sqrt{57} \approx 7.5498$

unit vec.: $\left\langle -\frac{2}{\sqrt{57}}, \frac{7}{\sqrt{57}}, -\frac{2}{\sqrt{57}} \right\rangle$

$\rightarrow \left\langle -\frac{2\sqrt{57}}{57}, \frac{7\sqrt{57}}{57}, -\frac{2\sqrt{57}}{57} \right\rangle$

Ex 5 a: $v = \langle 1, 5, 2 \rangle$, $w = \langle -6, 3, -2 \rangle$, $z = \langle 0, 5, -1 \rangle$

$\langle 3, 15, 6 \rangle - \langle -6, 3, -2 \rangle - \langle 0, 5, -1 \rangle \rightarrow \langle 9, 7, 9 \rangle$

No.

Date Apr 26 2021

8.5 Dot & Cross Products for Vectors

$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$

Ex 1: $u = \langle -1, 6, -3 \rangle$, $v = \langle 3, -1, -3 \rangle \rightarrow -3 + -6 + 9 \rightarrow 0$

Ex 2: $\underset{\leftrightarrow \sqrt{16+1+9}}{u = \langle -4, -1, -3 \rangle}$, $\underset{\leftrightarrow \sqrt{49+9+16}}{v = \langle 7, 3, 4 \rangle} \rightarrow -28 + -3 + -12 \rightarrow \cos^{-1}\left(\frac{-43}{\sqrt{26}\sqrt{74}}\right) \rightarrow 168.613°$

Cross Product:

$a = \langle x, y, z \rangle$

$b = \langle m, n, p \rangle$

$$a \times b = \begin{bmatrix} i & j & k \\ x & y & z \\ m & n & p \end{bmatrix}$$

$$= \begin{bmatrix} y & z \\ n & p \end{bmatrix} i - \begin{bmatrix} x & z \\ m & p \end{bmatrix} j + \begin{bmatrix} x & y \\ m & n \end{bmatrix} k$$

$$= (yp - zn)i - (xp - zm)j + (xn - ym)k$$

Ex 3: $u = \langle 6, -1, -2 \rangle$ $v = \langle -1, -4, 2 \rangle$

$$= \begin{bmatrix} i & j & k \\ 6 & -1 & -2 \\ -1 & -4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -2 \\ -4 & 2 \end{bmatrix} i - \begin{bmatrix} 6 & -2 \\ -1 & 2 \end{bmatrix} j + \begin{bmatrix} 6 & -1 \\ -1 & -4 \end{bmatrix} k$$

$$(-(2) - (-2 \cdot -4))i - (6(2) - (-2)(-1))j + (6(-4) - (-1)(-1))k$$

$$-10i - 10j + (-25)k \rightarrow \langle -10, -10, -25 \rangle$$